

Société de Calcul Mathématique SA  
*Outils d'aide à la décision*

Fédération Française des  
Jeux Mathématiques



**Mathematical Competitive Game 2017-2018**

*Distribution of Goods*

**Fédération Française des Jeux Mathématiques**  
*(French Federation of Mathematical Games)*

**and**

**Société de Calcul Mathématique SA**  
*(Mathematical Modelling Company, Corp.)*

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## I. Presentation of the Games

The "Mathematical Games", jointly organized by FFJM and SCM, have existed for eight years; the previous ones were:

- In 2008-2009, Conception of a bus transportation network in a city, in partnership with Veolia Transport;
- In 2009-2010, Conception of an electricity distribution network, in partnership with RTE (Réseau de Transport d'Electricité);
- In 2011-2012, Search for the best itinerary by a car, in partnership with the newspaper Auto Plus;
- In 2012-2013, Fighting forest fires in Siberia, in partnership with the Paris Firemen Brigade;
- In 2013-2014, Checking an industrial process;
- In 2014-2015, Uncertainties in GPS Positioning, in partnership with the French Institute for Transportation Science and Technology, Geolocalisation Team (IFSTTAR/CoSys/Geoloc) and the French Ministry of Transportation, Mission for Tarification Pricing (MEDDTL/DIGITIM/SAGS/MT);
- In 2015-2016, False Alarms in a Sensor Network, in partnership with the French IRSN (Institut de Radioprotection et de Sûreté Nucléaire);
- In 2016-2017, From the Earth to the Moon.

They deal with the resolution of a "real life" problem, that is a problem of general concern, but simplified in its mathematical contents. Still, the resolution typically requires several months of work.

Candidates may compete individually or as groups, for instance high school classes, or college students, or university students, preparing a "memoir" for the end of their studies.

Two categories of prizes are given:

### *Individual prizes:*

For the winner: 500 Euros

For the second: 200 Euros

For the next three: 100 Euros each.

### *Prizes for groups:*

For the winner: 500 Euros

For the second: 200 Euros

For the next three: 100 Euros each.

The total amount of prizes is therefore 2 000 Euros. The best solutions are published on the web site of FFJM, on the web site of SCM, and on the web sites of our partners. The official announcement of the results and the ceremony for prizes occur during the "Salon

de la Culture et des Jeux Mathématiques" (Fair for Mathematical Culture and Games), which is held in Paris, each year, during the month of May.

The winners, previous years, gained considerable recognition, both in the press and television in their respective countries.

## II. The 2017-2018 Game

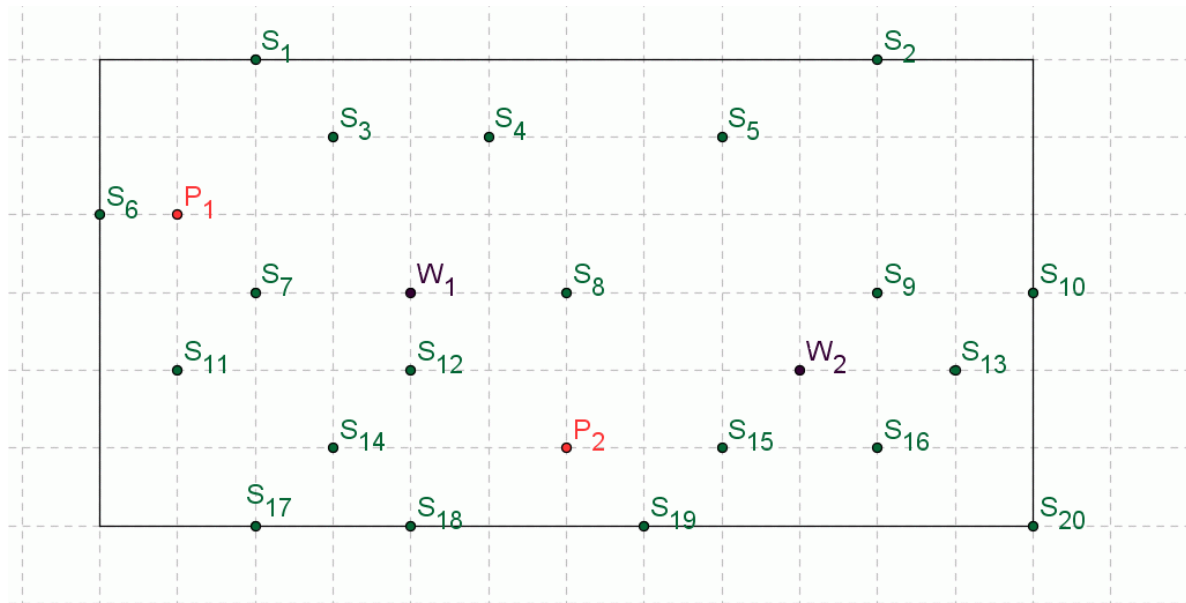
### A. General presentation of the subject

#### 1. The products

An Industry makes and sells two products : heaters and air-conditioners. Both products are seasonal, but the Industry expects that the sales will not occur at the same time, and thus the amount of work and sales should be rather constant, year round. The general question concerns the organization of the deliveries.

#### 2. Location of resources

The Industry has 2 plants, 2 warehouses and 20 shops, as presented on the following figure:



$P = \text{plant}, W = \text{warehouse}, S = \text{shop}.$

Each plant may produce any of the two products, and may switch from one to the other with no delay. The workers are qualified for both products with no difference. The warehouses and the shops can accommodate both products with no difference.

Each vertical or horizontal segment represents 1 km. In order to go from a point to another, one must follow these segments : there is no "shortcut".

### 3. Sales

Each shop is open six days a week, with no exception. The customer should find, in each shop, each day, at least one item of each product, otherwise he will be disappointed.

The file <http://www.scmsa.eu/archives/sales.xls> indicates the sales of each product, for each shop, over one year (2015). Sales are different from one shop to the other. These are recordings from the past, but one may consider that, for the new year, for each product, the monthly sales for each shop should be within  $\pm 10\%$  of the recorded data, for this shop and same period. Of course, the precise values of the expected sales are unknown.

Prices : the price of the heater is 100 Euros, the price of the air conditioner is 300 Euros (VAT not included).

### 4. Storage capacity

For each warehouse and each product, the storage capacity is fixed at 650 items. It has been computed so that it should be enough to represent one week of total sales (total for all shops), divided by 2 (since there are 2 warehouses). In other words, the capacity of the sum of both warehouses is the maximum of the sales over one week, for each product. This means in practice that the Industry expects to restock the warehouses on a weekly basis.

For each shop and each product, the storage capacity is the following:

Shop	P1	P2
1	15	15
2	15	15
3	20	20
4	20	20
5	15	15
6	20	20
7	30	30
8	30	30
9	35	35
10	25	25
11	30	30
12	30	30
13	30	30
14	35	35
15	30	30
16	40	40
17	25	25
18	20	20
19	15	15
20	20	20

It is intended to be the maximum of the daily sales for the reference year. This means in practice that the Industry expects to restock the shops on a daily basis.

Both warehouses capacities and shops capacities may be verified from the file "sales.xls".

These capacities are fixed and, due to practical limitations, cannot be expanded. So, in case of strong demand in a shop, the only resource is to have more deliveries from the warehouses.

## **5. Delivery tour**

The deliveries are done by trucks, from the factories to the warehouses and from the warehouses to the shops. Each heater has a volume (with package) of  $0.4 \text{ m}^3$  and each air-conditioner has a volume (with package) of  $0.8 \text{ m}^3$ . Each truck may take a total volume of  $20 \text{ m}^3$ ; weight is not a limitation here. The cost of delivery is 1 Euro per km (including the salary of the driver, fuel, and so on). Acquisition costs and maintenance costs for the trucks are not taken into account. The garages for the trucks are supposed to be inside the warehouses.

## **6. Complements and clarifications**

One may consider that the Industry has a new owner, who starts on January 1<sup>st</sup>, 2018. This new owner wants to redefine the logistics, with sole information the sales during the year 2015. We may assume that, on December 31<sup>st</sup>, 2017, everything is empty: no items in the warehouses or in the shops. This new owner will make a "delivery plan" for the forthcoming year, which means that he will define the delivery routes (one truck may connect several shops) and sign a contract with a Truck Agency, which will provide the necessary trucks and drivers for the whole year. This contract is a commitment: no matter what happens, no matter whether the trucks are needed or not, the Industry will have to pay the Truck Agency. If more trucks are needed, they can be obtained from the Truck Agency, but there is a 20% penalty over the normal price.

# **III. Questions**

The question is to organize a production planning, for both items, and a delivery planning, to the warehouses and then to the shops. This means:

## **1. Planning organization**

- Decide each day how many items should be produced, in each plant;
- Decide how many items should be kept in the warehouses;
- Decide on the delivery to each warehouse: what day of the week, and how many items ?
- Decide on the delivery each day from the warehouses to the shops: how many items ?
- Define the routes and schedules for the delivery trucks,

so as to ensure maximal expected profit for the Industry, over one year.

## 2. Probability law

What is the probability law of the expected profit, over one year ?

## 3. Comments

As it is usually seen in this type of question, the minimal cost is obtained in producing nothing and selling nothing. Here, we want to optimize the profit, not the cost. One aspect is unclear: if a customer does not find the products he wants, he is disappointed and the sales are missed. The participants should imagine a "penalty rule" in order to describe such a situation. Of course, this rule must be made very clear, because it has a significant influence upon the result.

Concerning the anticipated sales, we said in §3 above that they may differ from the observed sales by  $\pm 10\%$ . Thus, the participants may consider that, for each day, each product, each shop, the sales will be given by a uniform law, on the interval  $[-10\%, +10\%]$  centered at the observed sales.

The planning which is expected will be valid for the whole year. One does not expect any correction if, for instance, the sales proved very high during the first quarter.

## IV. Participation rules

The game starts on November 1st, 2017 and ends on April 30th, 2018. Prizes will be given in May 2018, during the "Salon des Jeux Mathématiques", in Paris.

Participants should send their solution, in pdf format, in English or in French, no later than April 30th, 2018, to the email address: **ffjm@wanadoo.fr**.

No preliminary registration is required. Everyone can participate.