Société de Calcul Mathématique SA
Outils d'aide à la décision

Fédération Française des Jeux Mathématiques

Mathematical Competitive Game 2016-2017

From the Earth to the Moon

Fédération Française des Jeux Mathématiques
(French Federation of Mathematical Games)

and

Société de Calcul Mathématique SA
(Mathematical Modelling Company, Corp.)

Comments, by Bernard Beauzamy

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I. Participation

There were 13 "individual" participations (resulting from 16 people) and 3 "group" participations, resulting from 17 people), so a total of 33 participants, from 6 countries: Bahrein, France, Great Britain, Portugal, Russia, USA

II. Results

A. *Individuals*

No First Prize

Second Prize: BIONDI Christophe, TRUCCHI Marco
and SIESS Vincent, ex aequo

Third Prize: HERSANT David

B. *Groups*

First Prize: DE ANGELIS Marco, GEORGES-WILLIAMS Hindolo, ROCCHETTA Roberto, SADEGHI Jonathan, INSTITUTE for Risk and Uncertainty, University of Liverpool.

No second and third prizes.

III. Comments about the solution

The result of the game is to indicate a probability (the probability to reach the target). So, quite clearly, this study is intended to help a decision. Typically, one may think that the work will be sent to the head of NASA, of the European Space Agency, or their equivalent in any country, in order to recommend that the projectile should be sent or not.

Unfortunately, the participants do not understand this properly. They start with technical information, angles, graphs, theorems, and so on, so the concrete result is very hard to find in their work, and sometimes the result does not appear at all: there is, instead, a lengthy discussion about solving differential equations by numerical methods, about software precision, and so on: discussion that the head of NASA would not read at all in any case.

Since the overall probability is very small (of the order of 36%), the report should say it very clearly, from the very beginning, and say quite explicitly "We do not recommend to launch the projectile".

Now, why is this so ? In order to help the people in charge of a decision make up their minds, we have to indicate what information we would need in order to improve the chances of success. In our case, a preliminary study (which has not been correctly done by many of the participants) indicates that the parameter α (which is unknown in the interval $2-2.5$) plays the leading role and that, if this parameter is too large, the projectile does not reach the Moon, but falls back on the Earth (the drag force is too important). So, our recommandation to the Head of NASA should be: we cannot launch until we have a better understanding about the value of α. So, before trying to send any rocket to the Moon, we should first understand the properties of the drag force for high velocities.

With these two items : do not launch, get more information on some parameter, the scientist has correctly fulfilled his duty towards the people in charge of the decision. Unfortunately, most scientists are unaware of this duty, and present their thoughts in their own language, which is of no help at all for the decision making people.

Let us now discuss the probabilistic aspects of the game. Probabilities here are used in order to represent the uncertainties about the values of some parameters. Several situations may occur :

- The measurement (for instance, mass of the Earth) is not completely precise ;
- The value of the parameter (for instance air density) changes all the time ;
- The physical law is unknown: this is the case for the parameter α.

Quite clearly, if all parameters were exactly known, it would be possible to determine the exact time of the launch. Please note that we have only one parameter at our disposal, which is the instant when we fire. All others are fixed, for instance the cannon is a hole in the Earth, and cannot be modified or moved: it is vertical.

We may try to give approximate values to each of the 15 unknown parameters, but if we give to each of them 10 possible equally spaced values, we have 10^{15} computations to make, which is not acceptable in practice.

A blind use of Monte Carlo method is not acceptable here. Assume for instance that success (that is, reaching the Moon) would be obtained only if all parameters were in their first tenth (the first $\frac{1}{10}$ of their variation domain). Then the probability of success would be $\frac{1}{10^{15}}$ and such a situation will never appear if one throws a few millions or billion runs.
For a more sophisticated and explicit example, please see the final example in
http://www.scmsa.eu/archives/BB_Wilks_2016_01_11.pdf

Another question, which is not mentioned in the game, is that of independence of parameters. If one underestimates the value of air density at some altitude, one is likely to underestimate the values at all altitudes, or at least the next ones.

So, the approach we recommend is a mix between physics and probabilities. One should at least understand the physical meaning of all parameters and the direction of their influence. For instance, if α increases, the drag force increases, and it becomes more difficult to reach the Moon, similarly for all air densities and the mass of the Earth. One should also try to solve the problem, giving to the main parameters their extremal value (in particular to α, of course).

When it is recognized that α is the main parameter, one should give to it a number of specific values, for instance 10 equally spaced ones (one may start with a smaller number, say 3 or 4). For each of these values, say α_k, $k = 1,...,10$, one has to solve a problem of conditional probabilities, namely : what is the probability to reach the Moon, assuming $\alpha = \alpha_k$; at this point, the use of a Monte Carlo method upon all other parameters is justified. The final probability is then obtained from all conditional probabilities.