Société de Calcul Mathématique SA
Outils d'aide à la décision

Fédération Française des Jeux Mathématiques

Mathematical Competitive Game 2016-2017

From the Earth to the Moon

Fédération Française des Jeux Mathématiques
(French Federation of Mathematical Games)

and

Société de Calcul Mathématique SA
(Mathematical Modelling Company, Corp.)

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I. Presentation of the Games

The "Mathematical Games", jointly organized by FFJM and SCM, have existed for eight years; the previous ones were:

- In 2008-2009, conception of a bus transportation network in a city, in partnership with Veolia Transport;
- In 2009-2010, conception of an electricity distribution network, in partnership with RTE (Réseau de Transport d'Electricité);
- In 2011-2012, search for the best itinerary by a car, in partnership with the newspaper Auto Plus;
- In 2012-2013, fighting forest fires in Siberia, in partnership with the Paris Firemen Brigade;
- In 2013-2014, checking an industrial process;
- In 2014-2015, uncertainties in GPS Positioning, in partnership with the French Institute for Transportation Science and Technology, Geolocalisation Team (IFSTTAR/Co-Sys/Geoloc) and the French Ministry of Transportation, Mission for Tarification Pricing (MEDDTL/DIGITIM/SAGS/MT);
- In 2015-2016, False Alarms in a Sensor Network, in partnership with the French IRSN (Institut de Radioprotection et de Sûreté Nucléaire)

They deal with the resolution of a "real life" problem, that is a problem of general concern, but simplified in its mathematical contents. Still, the resolution typically requires several months of work.

Candidates may compete individually or as groups, for instance high school classes, or college students, or university students, preparing a "memoir" for the end of their studies.

Two categories of prizes are given:

Individual prizes:
For the winner: 500 Euros
For the second: 200 Euros
For the next three: 100 Euros each.

Prizes for groups:
For the winner: 500 Euros
For the second: 200 Euros
For the next three: 100 Euros each.

The total amount of prizes is therefore 2 000 Euros. The best solutions are published on the web site of FFJM, on the web site of SCM, and on the web sites of our partners. The official announcement of the results and the ceremony for prizes occur during the "Salon de la Culture et des Jeux Mathématiques" (Fair for Mathematical Culture and Games), which is held in Paris, each year, during the month of May.

The winners, previous years, gained considerable notoriety, both in the press and television in their respective countries.

II. The 2016-2017 Game

A. General presentation of the subject

We refer to the novel "From the Earth to the Moon", by Jules Verne, 1865.
Please see: https://en.wikipedia.org/wiki/From_the_Earth_to_the_Moon

We recall that a cannon shoots a projectile to the Moon. The cannon is laid in a vertical hole, 300 m deep and 18 m wide, situated near Tampa, Florida, USA.
See :
https://en.wikipedia.org/wiki/Space_gun

B. Preliminary consideration

Everyone knows that sending a manned projectile to the Moon, using a cannon, is impossible, because of the initial acceleration, which would kill anyone inside. This argument is correct, but fairly incomplete. It seems to admit the possibility of sending an unmanned projectile. But, in fact, no matter what the initial velocity is, the air resistance is so strong that it will prevent the projectile from reaching any significant altitude. For instance, with the data given below (projectile of mass 1 000 kg), with initial velocity 12 km/s, the force due to gravitation is $9.8\times10^{3}\ N$ and the force due to air resistance is $5.5\times10^{8} N$; in such a case, the projectile would not go above the altitude of 1.4 km.

So a continuous action is needed, for instance by means of the thrust of a reactor, and a simple initial blast would never succeed. This was overlooked by Jules Verne.

People often have a wrong perception of this type of problem: they think that the primary forces are gravitational, whereas in fact the main force is air resistance. This makes a lot of differences in terms of uncertainties, as we will now see.

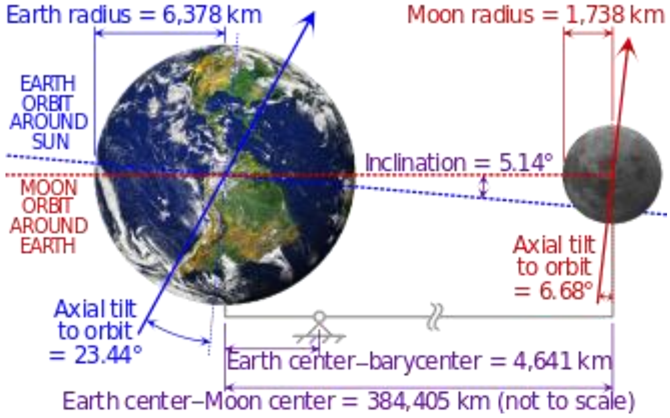
C. Technical data

We assume that mass of the projectile is exactly 1 ton (1 000 kg), with no error. This mass will be assumed to be invariant, for simplicity. The shape of the projectile is a sphere of 1 meter radius.

The projectile is subject to a constant thrust of $4\times10^{6} N$ during 30 seconds. There are no uncertainties upon the thrust, nor about the duration. We do not care about the structure of the cannon, and we do not care about the method which was used to obtain the thrust.

We consider that, at any time, the velocity is oriented towards the vertical, with no error.

The relative positions of the Earth and of the Moon are simplified, as shown in the picture below:



The orbit of the Moon is assumed to be a perfect circle, of radius 384,405 km, situated in the ecliptic plane. Therefore, the rotation speed of the Moon around the Earth is supposed to be constant.

The shooting was made at a time such that the projectile should hit precisely the center of the disk representing the Moon, should everything work as expected.

The projectile is subject to two gravitational forces only: Earth and Moon; all other gravitational forces are neglected. The Earth is supposed to be perfectly spherical, with radius 6 378 km and mass 6×10^{24} kg. The Moon is supposed to be perfectly spherical, with radius 1 738 km and mass 7.3×10^{22} kg.

The problem is to be solved in a system of coordinates linked to the Earth; we assume the Earth to be motionless; in particular, we ignore the rotation around the Sun and the rotation around itself. This way, the problem is a 2d problem, in the ecliptic plane, and the initial velocity ignores the component which results from the rotation of the Earth around itself.

For the density of the air, we will take the standard atmosphere data
(see for instance https://en.wikipedia.org/wiki/International_Standard_Atmosphere);
they are given in the table below:

Altitude [m]	Temperature [Kelvin]	Density [kg/m3]
0.00000	288.150	1.22500
5000.00	255.650	0.736116
10000.0	223.150	0.412707
15000.0	216.650	0.193674
20000.0	216.650	0.0880349
25000.0	221.650	0.0394658
30000.0	226.650	0.0180119
35000.0	237.050	0.00821392
40000.0	251.050	0.00385101
45000.0	265.050	0.00188129
50000.0	270.650	0.000977525

The air density will be considered as constant between two altitudes in the table above, and neglectible above the altitude of 50 000 m.

The formula for air resistance (drag) is supposed to be:
(see https://en.wikipedia.org/wiki/Drag_(physics))

$$F_D = \frac{1}{2}\rho V^{\alpha} C_D A$$

where :

F_D is the drag force;

ρ is the density of the air, given by the table above, in kg / m^3 ;

V is the speed of the projectile, in m / s ;

α is a coefficient ; one usually takes $\alpha = 1$ for very low speeds (sail boat), $\alpha = 2$ for higher speeds (a car, a plane). In the present case of extremely high speed (hypersonic velocity) in rarefied atmosphere, nobody knows exactly. So we will say that α may take any value, with uniform law between 2 and 2.5.

C_D is the drag coefficient ; for a sphere, the value is 0.5;

A is the sectional area of the sphere, in m^2.

III. Question

There is a unique question: What is the probability that the projectile hits the Moon ?

In order to solve this question, one has to make several assumptions about the origin of the errors, and their respective laws.

In this problem, we consider that all parameters which are under control have no uncertainty at all (duration of the thrust, value of the thrust, orientation of the velocity, and so on). This is not completely true, but will make the problem simpler.

Other parameters affected by uncertainties are in particular :

- Uncertainty about the precise position of the Moon when firing ;
- Uncertainty about the mass of the Earth ;
- Uncertainty about the mass of the Moon ;
- Uncertainties about the density of the atmosphere at different altitudes and the resulting forces ;
- Uncertainty about the value of the coefficient α in the formula above.

The participants may decide, for all these items, to chose any specific law for the uncertainties, for instance a uniform law around $\pm 10\%$ of the nominal value, or a gaussian law of proper variance.

Usually, one takes gaussian laws for parameters which are reasonably well-known (for instance the mass of the Earth) and uniform laws for parameters on which the knowledge is rather poor (for instance the coefficient α).

All choices should be clearly stated, since the result will depend upon the definition of these laws.

IV. Participation rules

The game starts on November 1st, 2016 and ends on April 30th, 2017. Prizes will be given in May 2017, during the "Salon des Jeux Mathématiques", in Paris.

Participants should send their solution, in pdf format, in English or in French, no later than April 30th, 2017, to the email address: **ffjm@wanadoo.fr.**

No preliminary registration is required. Everyone can participate.