Matlab PDE Toolbox: an error in the function calculating the shear strain

By Gottfried Berton, Adrien Schmitt, Alisson Stocchetti, SCM SA

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As part of a contract with the French National Radioactive Waste Management Agency (Andra), we had to model the mechanical behaviour of a concrete structure designed to receive nuclear wastes.

We used the finite element method in order to calculate the strains at each point of the structure. This method is widely used in civil engineering and allows solving the equations from continuum mechanics (structural model linear elasticity) by discretising the domain into finite elements.

During the execution of this work, we saw that the Matlab-2017b function "strain.exy" of the Partial Differential Equation Toolbox returns an erroneous result. We have highlighted this in three different ways:

- Comparing the result of Matlab with another software (RFEM);
- Recalculating the strains by hand and comparing the result with Matlab and RFEM;
- Checking the source code of Matlab to see how the strain is calculated.

Matlab returns the following elements for each node of a 2D mesh:

- the strain tensor;
- the stress tensor;
- the displacements in X and Y axis.

When comparing the results with the RFEM software, we see that the element γ_{xy} of the strain tensor of Matlab is very different from RFEM's. This shear coefficient represents the transformation from a rectangle to a parallelogram.

For both software, we use the following parameters in the simulations:

- Young modulus $E = 43{,}5\ \text{GPa}$;
- Poisson's ratio $\nu = 0{,}25$;
- Horizontal load 1,6 MPa;
- Vertical load 1,2 MPa;
- Forbidden vertical displacement at angles 0° and 180° ;
- Forbidden horizontal displacement at angles 90° and 270°.

The result for the RFEM software is displayed below:

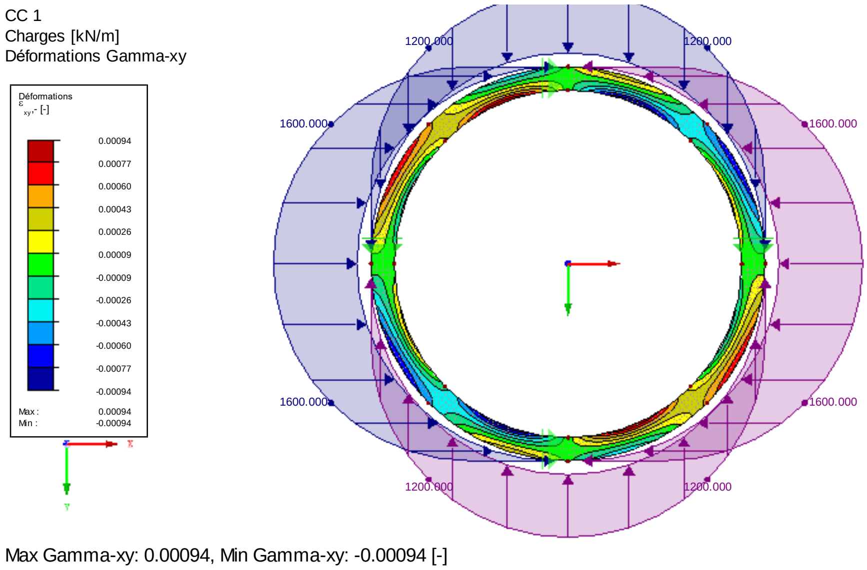

Figure 1 : shear strain obtained with RFEM

The result obtained with Matlab for the coefficient γ_{xy} is displayed below and is completely different:

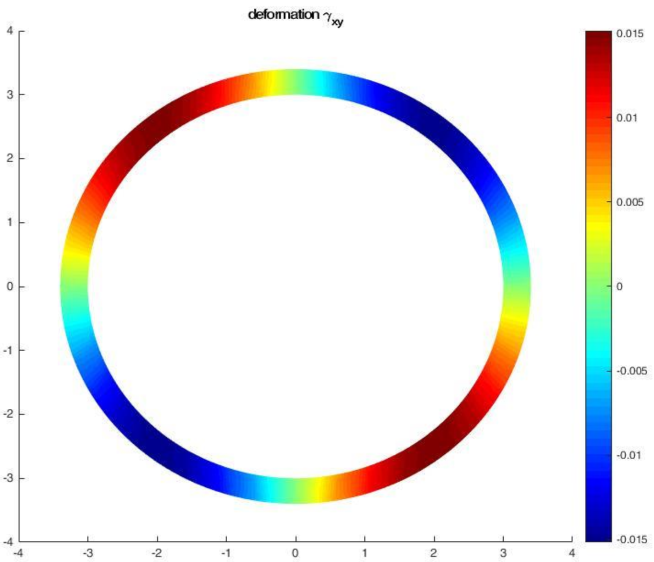

Figure 2 : shear strain obtained with Matlab

The values are between -6×10^{-3} and 6×10^{-3} for Matlab, while RFEM returns values between -9×10^{-4} and 9×10^{-4}. Moreover, for RFEM the shear strain varies with respect to the ray, whereas the strain is constant in function of the ray for Matlab.

In order to distinguish, between Matlab and RFEM, which software returns an erroneous result, we calculate the shear strain by ourselves from the shear stress σ_{xy} returned by Matlab (the stress tensor is the same for Matlab and RFEM, so it does not matter which software has been used here) using the following equation:

$$\gamma_{xy} = \frac{2(1+\nu)}{E}\sigma_{xy} .$$

When calculating by ourselves the coefficient γ_{xy} using this formula, we obtain the same result as RFEM and a different result from Matlab. Thus, the coefficient γ_{xy} returned by Matlab is incorrect.

The figure below shows the result when calculating the shear strain by ourselves:

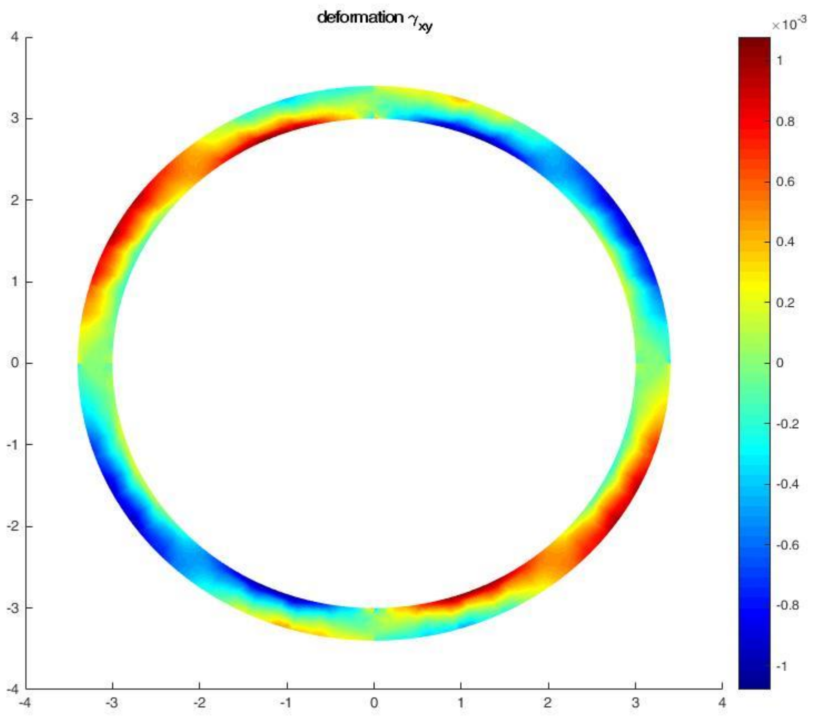

Figure 3 : shear strain recalculated from the stress tensor

After verification of the source code of Matlab that calculates the shear strain (\toolbox\pde\+pde\@StaticStructuralResults\StaticStructuralResults.m), it appears that Matlab calculates the coefficient γ_{xy} with the following equation, which is wrong:

$$\gamma_{xy} = \frac{\partial u_y}{\partial x} .$$

One should instead use the following equation:

$$\gamma_{xy} = \frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} .$$

The other components of the strain tensor are calculated properly.

From a general point of view, one should be careful when using such black-box functions because they may give erroneous results, and lead to wrong decisions.

gottfried.berton@scmsa.eu
adrien.schmitt@scmsa.eu
alisson.stocchetti@scmsa.com

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May 4th, 2018 : Matlab confirms the bug :

Ravi Kumar answered your question I found an error in the "strain.exy" function from PDE toolbox, can someone confirm?

Hello Gottfried,

I confirmed that this is a bug. This will be fixed in a upcoming release. I am sorry of the inconvenience and extra work this has caused you. Thanks for sharing the detailed validation report.

Regards ,
Ravi