Research Program:

Malfunctions in Sensors' Networks

The detection of a radioactive cloud

redaction : Veronika Khalipova, Guillaume Damart, Bernard Beauzamy

version : 2015/11/20

I. Introduction

Nowadays, nuclear plants are an important source of electricity. There are 191 plants worldwide, 19 in France. France has the second place in the world in terms of production of electricity by nuclear plants. Although these plants have many advantages, there were a small number of accidents.

The most significant ones occurred in Three Mile Island, USA (1979), Chernobyl, Ukraine (1986), in Fukushima, Japan (2011). During these accidents, radionuclides were released in the environment.

Many countries (in particular France and Ukraine) perform a constant monitoring of radioactivity in the atmosphere, by means of a network of sensors. If a dangerous level is detected, people can be warned and evacuated.

Our research program "Malfunctions in Sensors Networks" deals with the propagation of radionuclides in the atmosphere and their possible detection by a network of sensors, taking into account all possible malfunctions of the network:

- uncertainties ;
- failures ;
- false alarms.
The objectives of the research are:

1. To check the effectiveness of a given network of sensors: is it possible to reconstruct the shape of the cloud, from the indications given by the sensors and with what accuracy?

2. To design more robust network of sensors, which will be more resistant to uncertainties, failures and false alarms.

For this, we will make various assumptions upon the type of cloud, its direction, speed, and so on. In each case, we will see what kind of detection occurs.

In order to investigate a large number of scenarios, we developed a simulator in Matlab, in which all parameters can be changed.

II. Basic situation and its description

A. Theory

In the simplest situation, we have only one sensor and the cloud moves with a uniform motion, with no modification of its shape and concentration. This may look quite simple, but already in this case the problems connected with uncertainties, failures and false alarms will appear quite clearly.

First of all, we have chosen an approach in which we “pixellise” the cloud. In such an approach, we do not give any specific geometrical shape to the cloud. On each pixel of the domain, we have a radioactivity level, which is a real number. We certainly lose in terms of precision, because the cloud does not have specific boundaries anymore, and we always have the usual difficulties connected with pixels: all points inside a given pixel are treated the same way.

Compared to the presentation of the cloud as a collection of geometrical shapes with different levels of radioactivity, our approach is closer to reality. In fact, there is always a natural level of radioactivity (which may differ from one zone to another) and radioactive clouds do not have precise boundaries. A pixel of 1 x 1 km seems reasonable, in terms of precision.

Also, the administrator of the network of sensors does not know anything about the shape of the cloud. He has at his disposal only the information given by the sensors, plus some meteorological information (speed and direction of wind).

In this preliminary simulation, the radioactive cloud moves along a straight line from East to West: entrance point $A$ and exit point $B$. The cloud will have the shape of a disk of radius $r$. 
We denote by $V$ the speed of propagation; in the numerical examples below, we will take $V = 10$ km/h.

Let $\Delta$ be the length of the segment $AB$ and let $C$ be the center of the disk representing the radioactive cloud. The equation of the movement of $C$ is:

$$AC = Vt \frac{AB}{\Delta}$$

if we take the origin of time ($t = 0$) at the moment where $C$ was in $A$.

**B. The domain**

Since the shape of the domain has no importance, we take a simplified domain, namely a square. The dimensions of the square will be comparable to the dimensions of continental France, namely $750 \times 750$ km. The domain is thus divided into 562 500 pixels, each of them being $1 \times 1$ km. The origin of the axes is the point left bottom of the square.

**C. Pixel representation of the radioactive cloud**

Radioactivity may be anywhere in the domain, so each pixel has its own level of measurement, represented by a matrix $ral(x, y)$, which characterizes the radioactivity level in this pixel. If there no radioactivity, its value is 0 (no radioactivity) in the other case, it is 1 (high level). Here, we have a simplified representation of the radioactivity; real values will be introduced later in the project.

**D. A unique sensor**

A unique sensor, denoted by $U$, is put somewhere in the square area and its coordinates are $(x_U, y_U)$; it measures the radioactivity level in the pixel where it is located, that is the value of $ral(x_U, y_U)$.

**E. Time units**

The sensor monitors the environment continuously, but transmits the information only every ten minutes. In other words, we will consider that all motions are discretized and viewed every 10 minutes. The time unit (TU) is denoted by $\tau$; it may be modified in the simulator.


**F. The software**

**1. Input variables**

In the software, we have as input variables:

- time unit;
- coordinates of the sensor;
- velocity of propagation of the cloud;
- radius of the radioactive cloud;
- coordinates of the entrance point;
- coordinates of the exit point.

The software simulates the movement of the radioactive cloud and its detection by the sensor.

**2. Output variables**

The outputs are:

- the fact whether the radioactive cloud was detected or not;
- the intervals of detection time;
- the estimated radius of the cloud.

**3. Mathematical approach and computer implementation**

First, the software finds the roots of a system of equations, and obtains the coordinates of the center of the cloud at first and last detection. Using these data, a cycle of ‘while’ is performed until the first and then last detection coordinates are reached by the cloud or while the detection time is less than the maximum time. The times of first and last detection are used in the calculation of the estimated size of the cloud.

The program represents the radioactive cloud as a collection of pixels. The coordinates of its center $x_c, y_c$ are determined from the law of uniform linear motion.

We denoted by $n$ the number of time units; at each TU, $n$ is increased by 1. The time will be written as:

$$T = \tau \cdot n \quad (2)$$

The array $ral(i, j, n)$ is filled with ‘1’ for the high radioactivity level and in the other case with ‘0’.
At each step of the cycle 'while', the circle matrix array is filled with 1, and at each step it moves. For each step, a new matrix $ral(i, j, n)$ is created and all information is saved.

**G. Evaluation of the size of the cloud**

In order to evaluate the size of the cloud, we need to know the times of first and last detection, that is when the sensor sees the cloud for the first time and last time. Here, we must distinguish carefully between the "true" detection time (what the sensor sees) and the "announced" detection time (when the sensor speaks), because the sensor communicates only each ten minutes.

When there is a detection, it means that the disk of center $C$ and radius $r$ contains the sensor $U$. In other words, the initial and final positions of $C$, respectively denoted by $C_1$ and $C_2$ will be simply the intersections of the segment $AB$ with the circle of center $U$ and radius $r$ : see fig. 1 below.

So the coordinates of $C_1$ and $C_2$ will be the solutions of the system of equations :

$$\begin{cases} (x_U - x)^2 + (y_U - y)^2 = r^2 \\ y - y_A = \frac{y_B - y_A}{x_B - x_A} (x - x_A) \end{cases}$$

(3)

A solution exists only if $d(U, AB) \leq r$, and it is unique if $d(U, AB) = r$.

The coordinates deduced from this system of equations are used for the detection of the cloud. When the center of the cloud reaches the coordinates of first detection, the time of first detection is found. When it reaches the coordinates of last detection, then the time of last detection is found. From (1), we deduce $AC = Vt$, so the time of first detection, $t_1$, is given by the formula :

$$t_1 = \frac{AC_1}{V}$$

(4)

and similarly :

$$t_2 = \frac{AC_2}{V}$$

(5)
Here, \( t_1, t_2 \) are "true" times of detection. Now, the sensor announces this detection with a delay, which is at most \( \tau \). The announced times of detection, denoted respectively by \( T_1, T_2 \), satisfy:

\[
\begin{align*}
    t_1 & \leq T_1 \leq t_1 + \tau, \quad t_2 \leq T_2 \leq t_2 + \tau \\
\end{align*}
\]  

This gives:

\[
\begin{align*}
    T_2 - T_1 - \tau & \leq t_2 - t_1 \leq T_2 - T_1 + \tau \\
\end{align*}
\]  

If the sensor detected the cloud, an estimate for the diameter of the cloud is:

\[
Diam = V \left( T_2 - T_1 \right) \pm V \tau
\]  

We observe here that we could find the diameter of the cloud (up to some uncertainty) because we assumed that it moved on the known segment \( AB \).

If we know only the instants \( T_1, T_2 \) and the speed \( V \), then the radius of the disk is not properly determined, as we see on this picture:
In this picture, all segments $C_1C_2$ have same length, and $UH$ is the bisector of the segment $C_1C_2$. However, these segments may be at any distance from $U$. The radius of the disk is $UC_1$ in each case.

So, the conclusion in this case is that our problem is not correctly posed: we cannot find the radius of the disk from the information given by a single sensor.

**H. Results**

For the simulations, we take different position of the sensor (on the line AB and far from the line), also different values for the radius of a cloud.

1. **First simulation**

In a first case, we take the values:

- $x_a = 750$ km, $y_a = 500$ km,
- $x_b = 0$ km, $y_b = 100$ km,
- $x_u = 400$ km, $y_u = 310$ km,
- $r = 50$ km, $v = 10$ km/h, $\tau = 10$ min.

Then the total time is $t_m = 85$ [h], and number of time units is $K = 510$. Then the difference in true times of detection satisfies:

$$9.83 \leq t_2 - t_1 \leq 10.16 \text{ (h)},$$

and the radius satisfies:

$$r_{c_1} = 49.17 \leq r \leq r_{c_2} = 50.83 \text{ (km)}$$

2. **Second simulation**

In the second case, the value of the radius is twice as big as in the first case, that is $r = 100$ km.

Then the difference in true times of detection satisfies:

$$19.83 \leq t_2 - t_1 \leq 20.16 \text{ (h)},$$

and the radius satisfies:
\[ r_c = 99.17 \leq r \leq r_c = 100.83 \text{ (km)} \]

Fig. 3 - First and last time of detection of the radioactive cloud (50 km)

Fig. 4 - First and last time of detection of the radioactive cloud (100 km)

Fig. 3 shows the radioactive cloud at first (right circle) and last time of detection (left circle), with radius of the cloud equal to 50 km, and the position of the sensor.

Fig. 4 shows the radioactive cloud at first and last time of detection, for a radius of 100 km, and the position of the sensor.
3. Third simulation

In a third case, the sensor's position is changed: the sensor is at distance 80 km from the line.

Then the difference in true times of detection satisfies:

\[ 13.5 \leq t_2 - t_1 \leq 13.83 \text{ (h)} \]

and the radius satisfies:

\[ r_{c_1} = 67.5 \leq r \leq r_{c_2} = 69.16 \text{ (km)} \]

The positions of the radioactive cloud at first and last time of detection and the position of the sensor are presented on Fig.5.

As we already said, this figures shows clearly that we cannot find the radius of the disk from the information given by a single sensor. For a given radius, the further the cloud is, the shorter the interval of detection will be.

III. Failure of the sensor

A. Theory

In the previous paragraph, we analyzed the situation when the sensor works correctly; let us consider now the case when it may present some malfunctions.

The first one will be a failure of the sensor, which means that it transmits nothing at all. This is a very common situation for all types of sensors.
Since we assume that our sensor emits every 10 minutes (our "Time Unit"), the administrator should always receive a signal at each TU. If he receives nothing, this means that the sensor does not work, which will be recorded by "F" (fail) on the time series.

We use the observations made upon the existing TELERAY network, the following way:

There are 714,656 daily observations, out of which 10,154 show a very low radioactivity level (below 40 nS/h), which we consider as impossible (this is below the normal radioactivity level). We interpret that as a "failure" of the sensor, and we attribute a value to this probability, by the formula:

\[ p_{BR} = \frac{10154}{714656} \approx 0.014 \]

This value may be modified in the simulator, and it will be used in the numerical simulations below.

In order to compute the probability of failure on the interval AB, we take into account the fact that the sensor worked correctly before failure and make the assumption that the probability of failure does not change with time. Then the probability to break at least once during the movement of the cloud from A to B is

\[ \text{Prob}(\text{failure on the segment } AB) = 1 - (1 - p_{BR})^K \]  

(9)

If the sensor fails, it is repaired. This takes time \( T_R \); we denote by \( T_R \) the time for repair. In numerical simulations, we take the value \( T_R = 24 \) h, that is \( T_R = 144 \tau \).

**B. Software implementation**

In the software implementation, we use a random value with uniform probability on the interval [0 1]. The software gives failure if this value is less than the probability of failure at each time unit; in this case, it skips 144 time units so that \( n \) increases by 144.

If a failure occurs, then the matrix at this time \( ral(i, j, n) \) is assigned the value «/». For the time \( \tau \cdot (n+144) \), \( ral(i, j, n) \) is filled with 1 and 0 due to the radioactivity level and from this moment the cycle resumes with the usual increasing of \( n \). The software returns 'K' that is correct work if the random value is less than the probability of failure at each TU and the sensor works correctly, \( n \) increases by 1, array \( ral(i, j, n) \) is filled with «1» or «0» at each TU.
C. Results

First, we consider the value of probability of failure on the segment $AB$. We take the values as in the previous examples. Then $t_m = 85 \text{ [h]}, K = 510$. The probability of failure on the segment $AB$ is:

$$\text{Proba (failure on the segment } AB) = 0.999,$$

which is a large value. We observe here a very important fact: if the rates of failure are those of the present TELERAY network, almost certainly a failure would occur in the detection of a radioactive cloud, by any given sensor, when this radioactive cloud moves from one boarder to the other.

The TELERAY network is made of two kinds of sensors: ancient and recent. Let us compute separately the rate of failure for both.

- Ancient sensors: 9 444 failures upon 620 350, that is 0.015;
- Recent sensors: 710 failures upon 94 306, that is 0.0075.

So the IRSN, in order to improve reliability, should replace as many ancient sensors by recent ones. But, even doing so, the probability of no failure on $AB$ would be only 0.03, which is too small. The use of several sensors is clearly required.

We modeled different situations using the simulator.

1. First simulation

Let us consider the first case, when the failure occurs before the first detection. Since the sensor misses 24 hours due to repair, then it could miss 240 km of the cloud at most.

Let us consider than the sensor is situated on the line $AB$ and the size of the cloud is 100km.

Then the difference in true times of detection satisfies:

$$13.33 \leq t_2 - t_1 \leq 13.66 \text{ (h)},$$

and the radius satisfies:

$$r_c_1 = 66.67 \leq r \leq r_c_2 = 68.33 \text{ (km)}$$

In this case, the failure occurred before the first detection, so the estimated radius is less than normal. Compared to the normal case (the values of the estimated radius are be-
tween \( r_{c_1} = 99.17 \text{ km} \), \( r_{c_2} = 100.83 \text{ km} \), in the considered case the estimated radius is roughly 1.5 times less.

![Diagram](image)

Fig. 6 - First and last time of detection of the radioactive cloud when failure occurs before first detection

2. Second simulation

Using the same values, let us consider a second case, in which detection does not occur at all:

![Diagram](image)

Fig. 7 - Detection of radioactive cloud does not occur

For instance, if we have a cloud with diameter 100 km, and if the sensor breaks just before the cloud reaches it, the sensor misses all the cloud.

3. Third simulation

Again with the same values, let us consider a third case: we have a first and last detection with normal work of sensor, but the sensor breaks after the last detection. This does not influence the results: we do not miss any information about the radioactive cloud.
4. Fourth simulation

In the fourth case, the sensor breaks after detecting some part of the cloud. In this case, we cannot give a true value for the radius. Our estimate is smaller than reality. Then the difference in true times of detection satisfies:

\[ 4.33 \leq t_2 - t_1 \leq 4.66 \text{ (h)}, \]

and the radius satisfies:

\[ r_{c_1} = 21.67 \leq r \leq r_{c_2} = 23.33 \text{ (km)} \]

Compared to the normal case (the values of the estimated radius are between \( r_{c_1} = 99.17 \text{ km, } r_{c_2} = 100.83 \text{ km} \)), now, the radius is roughly 5 times less than in the normal situation; we miss a lot of information about the radioactive cloud.

D. Stationarity of the law

As we saw earlier, we assumed the law of the failure of a sensor, to be "stationary", which means that the probability of failure at a given time, knowing it worked previously, is always the same. This may seem to be restrictive, in the sense that aging must be taken into account.
In this case, the failure probability will increase due to aging at each time unit $\tau$. But the changes are very small. Our computations (which are not reproduced here) show that the probability of failure on AB will be close to the one obtained in the constant case (0.99938 or 0.99949).

From the data which have been recorded, for instance on the IRSN’s TELERAY network, we see that the probability of complete failure on the segment AB is almost 1, that is a big value. The same conclusion holds if we include an increase of the probability of failure due to aging.

IV. False alarm

A. Theory

The next type of malfunction is a false alarm. It is the situation where the sensor gives an alert, whereas there is no radioactive cloud in reality. The most likely cause for the false alarm is a component failure. False alarms may be caused by minor fluctuations in background radiation levels. The sensor can overestimate the true value of the radioactivity. False alarms are very costly and we must take this malfunction into account.

In this case, the sensor transmits the value “1” (high radioactivity level) but there is “0” (low radioactivity level) in fact. When it gives an alarm, someone must go and check whether it is true. When they find out that the alarm is false, then it takes a time denoted as $T_F$ in order to repair the sensor. In our simulations, we take $T_F = 12$ h. The sensor in this case will give an alarm for some period of time until it is disconnected, and then repaired.

For the operator of the network, it is harder to recognize a false alarm than a failure, since he receives the value “1” and does not know whether it correct or not, until an additional operation of validation of alarm is made.

The same as for the failure, we calculate the probability of false alarm for each time unit and on the segment AB. From the TELERAY observations, we compute a probability of false alarm $p_{FA} = 0.28 \cdot 10^{-3}$. Indeed, we have 194 measurements which indicate a radioactivity level above 1000 nS/h, for a total of 714 656 measurements.

The probability to give a false alarm on the interval AB is

$$\text{Proba (false alarm on the segment AB)} = 1 - (1 - p_{FA})^K$$ (10)
B. Software implementation

In the software implementation, we use a random number $s$ with uniform probability on the interval $[0 \ 1]$, and if its value is less than the probability of false alarm at each time unit, then the false alarm occurs. A false alarm may lead to the investigation of a radioactive cloud which does not exist in reality; the estimated size of the cloud (due to time of repair for the sensor) is 120 km. When the sensor gives a false alarm, it can be that it detects a false cloud with diameter 120 km.

If the false cloud occurs immediately before the first detection or after the last detection of the real cloud, the time of false alarm adds to the time of real cloud detection.

C. Results

First, we consider the value of the probability of false alarm on the segment $AB$. We considered the position of the sensor and the size of the cloud as in the previous examples. The probability of false alarm on the segment $AB$ is

$$Proba (false \ alarm \ on \ the \ segment \ AB) = 0.133.$$ 

We modeled different situations using simulator.

1. Simulation 1

Let us consider the case: false alarm but there is nothing.

We take the values as in the examples with the failure.

In this case, we do not take into account the time of false alarm, since we know that the real cloud does not exist. The size of the false cloud will be $d_{F_A} = 120$ [km]
2. Second simulation

Let us consider the case: false alarm before first detection of the real radioactive cloud. The values are the same as for the first case.

\[ \frac{12}{156.67 \leq r \leq 158.33} \text{ (km)} \]

Compared to the normal case (the values of the estimated radius are between \( r_{C_1} = 99.17 \) km, \( r_{C_2} = 100.83 \) km), in considered case the radius is bigger than the normal one by 57 km. So we overestimate the size of the cloud.

3. Third simulation

Let us consider the case: false alarm after last detection of the real radioactive cloud.
In this case, the time of false alarm is added to the time of detection of the real cloud. Then the radius satisfies:

\[ r_{c_1} = 139.17 \leq r \leq r_{c_2} = 140.83 \text{ (km)} \]

Compared to the normal case (the values of the estimated radius are between \( r_{c_1} = 99.17 \text{ km}, \ r_{c_2} = 100.83 \text{ km} \)), the radius is bigger than the normal one by 41.67 km. So we overestimate the size of the cloud.

False alarms may also occur before or after true alarms. Due to the false alarms, we can detect the real radioactive cloud too early or continue to detect it after it is gone. In these situations, we will overestimate the size of the real cloud.

### D. Failure and false alarm

A situation which can also be considered is an occurrence of failure and false alarm on the interval AB. They can be in different sequences (failure before false alarm or conversely), so there are different situations for malfunctions.

First, let us consider the probabilities of the different situations.

We denote the probability of correct work of the sensor at each time unit by \( p_{OK} \).

The probability that the sensor breaks at least once during movement of the cloud on AB, but does not give false alarm, is:

\[
\text{Proba (failure on the segment AB without false alarm)} = \sum_{k=1}^{K} p_{OK}^{k-1} p_{BR} = \frac{1 - p_{OK}^K}{1 - p_{OK} p_{BR}} \tag{11}
\]

The probability that the sensor gives a false alarm but does not break on the segment AB is:

\[
\text{Proba (false alarm on the segment AB without failure)} = \sum_{k=1}^{K} p_{OK}^{k-1} p_{FA} = \frac{1 - p_{OK}^K}{1 - p_{OK} p_{FA}} \tag{12}
\]

The probability that the sensor breaks and then has a false alarm on the segment AB is:

\[
\text{Proba (false alarm and failure before it on AB)} = \sum_{k=1}^{K-144} \sum_{m=1}^{k+1} p_{OK}^{k-1} p_{FA} p_{OK}^{m-1} p_{BR}
\]

\[
= \left( \frac{1 - p_{OK}^{K-144}}{1 - p_{OK}^{K-144}} - p_{OK}^{K-144} \cdot (K-144) \right) p_{FA} \cdot p_{BR} \tag{13}
\]

The probability that the sensor gives a false alarm and breaks after it on the segment AB is:
$Proba\ (false\ alarm\ and\ failure\ after\ it\ on\ AB) = \sum_{k=1}^{K} \sum_{m=1}^{K-k+1} P_{OK}^{k-1} p_{FA} p_{OK}^{m-1} p_{BR}$

\[= \left( \frac{1-p_{OK}^{K}}{1-p_{OK}} - p_{OK}^{K} \cdot K \right) \frac{p_{FA} \cdot p_{BR}}{1-p_{OK}} \]  \hspace{1cm} (14)

E. Numerical simulation

First, we compute the values of the probability of false alarm on the segment $AB$. We take the position of the sensor and the size of the cloud as in the previous examples. We have:

$Proba\ (failure\ on\ the\ segment\ AB\ without\ false\ alarm) = 0.979$

$Proba\ (false\ alarm\ on\ the\ segment\ AB\ without\ failure) = 0.0196$

$Proba\ (false\ alarm\ and\ failure\ before\ it\ on\ the\ segment\ AB) = 0.0186$

$Proba\ (false\ alarm\ and\ failure\ after\ it\ on\ the\ segment\ AB) = 0.0191$

As we can see from the results for the TELERAY case, the probability for both false alarm and failure are almost identical either the false alarm appears first or the failure. The false alarm is rather unlikely, and the failure is quite likely.

F. Detailed Results

1. False alarm and failure before it

The first type of situations is when a failure occurs first, and then a false alarm later.

Let us consider the first case: failure before first detection and false alarm after last detection.

The sequence of events in this case is: correct work for some time units (min), then failure, loss of $T_{k}$ time units, correct work for some time units and false alarm. There can be no time units with normal work before failure and after repair. Then a failure could occur at the first time unit and a false alarm right after repair.
So a failure could appear at $n=1,2,\ldots,K-145$ and after it a false alarm occurs at $n=146,147,\ldots,K$.

![Time diagram in case when failure occurs before first detection and false alarm after failure](image1)

**Fig. 13** - Time diagram in case when failure occurs before first detection and false alarm after failure

For example, a failure could occur before the first detection and a false alarm after last detection. Then the radius satisfies:

$$r_{c_1} = 83.33 \leq r \leq r_{c_2} = 84.99 \text{ (km)}$$

![Time diagram in case when failure occurs before first detection and false alarm after last detection](image2)

**Fig. 14** - Time diagram in case when failure occurs before first detection and false alarm after last detection

Compared to the normal case (the values of the estimated radius are between $r_{c_1} = 99.17$ km, $r_{c_2} = 100.83$ km), in the considered case the radius is smaller than the normal one by $15.84$ km due to the failure that occurred before the first detection (which occurred later). But also, the estimated value is not too small, due to the false alarm after the last detection, when the radius was overestimated.

![The position of the radioactive cloud after failure, then after last detection and false cloud before detection (dashed line)](image3)

**Fig. 15** - The position of the radioactive cloud after failure, then after last detection and false cloud before detection (dashed line)
In general, we have $T_f$ of false alarm that increases the estimated diameter of the cloud and $T_r$ of failure that decreases the value of estimated diameter. Then we have 12 hours at most that will decrease the value of estimated diameter:

$$d_{c_1} = 2r_{c_1} - 120, \quad d_{c_2} = 2r_{c_2} - 120 \text{ (km)}$$

Let us consider the second case: failure and then false alarm before the first detection.

In order to see this case we changed the position of the sensor so that more time units needed to reach it by cloud.

If a failure occurs before the first detection and then a false alarm right before the first detection, then we do not miss any information about the real cloud due to the failure. Also, we take into account the time of false alarm, since the detection of real cloud starts later. Then the radius satisfies:

$$r_{c_1} = 157.50 \leq r \leq r_{c_2} = 159.16 \text{ (km)}$$

Compared to the normal case (the values of the estimated radius are between $r_{c_1} = 99.17$ km, $r_{c_2} = 100.83$ km), in this case the radius is bigger than normal by 58.33 km due to the false alarm.

A failure before the first detection does not change the estimated size of the cloud, but a false alarm before the first detection could increase it by 120 km. Then the diameter of the cloud is between:

$$d_{c_1} = 2r_{c_1} + 120, \quad d_{c_2} = 2r_{c_2} + 120 \text{ (km)} \quad (15)$$

Let us consider the third case: failure right before first detection and false alarm after last detection. In this case we do not detect the real cloud due to the failure at all, but then we detect a false cloud. Since there is no real cloud when the false alarm starts,
then we do not take the time of false alarm into account and the estimated size of radioactive cloud is zero.

Fig. 17 - The position of the real cloud before failure and false cloud after last detection (dashed line)

2. The sensor gives a false alarm and breaks after it

Another type of situation is when the sensor gives false alarm and breaks after it.

If the sensor gives a false alarm, it can break just after it. The sequence of events is:

- correct work for some time units;
- false alarm;
- failure.

So a false alarm may appear at $n=1,2,...,K-1$ and after it a failure occurs at $n=2,3...,K$.

Fig. 18 - Time diagram in case when failure occurs before first detection and false alarm after failure

1. Case 1

Let us consider the case: false alarm before first detection and failure during detection.

Then the radius satisfies:

$$ r_{c_1} = 89.17 \leq r \leq r_{c_2} = 90.83 \text{ (km)}.$$
Compared to the normal case (the values of the estimated radius presented on the screen $r_c = 99.17$ km, $r_c = 100.83$ km) in this case the radius is smaller than normal by 10 km due to the failure during detection (not the whole cloud was detected). But also the estimated value is not too small due to the false alarm before first detection, when the radius was overestimated.

![Fig. 19 - The false cloud before first detection (dashed line) and cloud before and after the failure of a sensor](image)

A false alarm right before the first detection increases the diameter of the cloud by 120 km, but a failure during detection decreases the diameter by 240 km. We estimate the diameter of the cloud:

$$d_{c_1} = 2r_{c_1} - 120, \quad d_{c_2} = 2r_{c_2} - 120 \text{ (km)} \quad (16)$$

2. Case 2

Let us consider the case: false alarm before first detection and failure after last detection.

Then the radius satisfies:

$$r_{c_1} = 155.00 \leq r \leq r_{c_1} = 156.66 \text{ (km)}$$

Compared to the normal case (the values of the estimated radius presented on the screen $r_c = 99.17$ km, $r_c = 100.83$ km) in the considered case we overestimate the radius by 55.83 km due to the false alarm. A failure occurring after the last detection does not change the estimated value of radius.
Fig. 20 - The false cloud before first detection and last detection of the real cloud

A failure after the last detection does not change the estimated size of the cloud, but a false alarm before the first detection increases it by 120 km:

\[ d_{FA_1} = 2r_{C_1} + 120, \quad d_{FA_2} = 2r_{C_2} + 120 \text{ (km)} \]  

If we have a false alarm and a failure after the last detection, it does not influence the estimated size of the real cloud.

**G. Conclusions**

The probability to have a false alarm and then a failure, or conversely, is very small, but if this situation occurs, in most cases these two malfunctions compensate a little the result of their mistakes. The failure decreases the size of the estimated radius of the radioactive cloud, and the false alarm increases it.

**V. General conclusions**

We considered different situations which can occur in reality: the sensor works correctly, gives a false alarm, has a failure, or both malfunctions. The estimated size of the cloud has some uncertainty due to the delay in announcement. The size of the estimated radius will be more precise if the sensor is placed on the line of propagation of the radioactive cloud and the mistake will be greater when the distance from AB increases.

Most situations with failure or false alarm provide mistakes in the estimate of the size of the cloud, usually increasing the uncertainty. The situation with both failure and false alarm also changes the estimated size from its normal value.