Société de Calcul Mathématique, S. A.
Algorithmes et Optimisation

Analyse probabiliste de la séismologie

Mise en œuvre d'une méthode probabiliste,

comparaison avec la méthode déterministe

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Introduction

In this report, we show how to construct the probability law for the acceleration of earthquakes, inside a certain interval of magnitude and a certain interval of epicentral distance (that is a distance between “zone source” and “zone cible”), using only historical data. This means that we do not rely upon a "parametric" description : for instance, we do not assume that the law of attenuation is "log-normal". We construct it from scratch.

We treat several examples : one global and two specific ones. We show that the probabilistic laws that we build do not necessarily coincide with the usual attenuation laws, constructed from a log-normal function.

The usual method, called PASH (Probabilistic approach to seismic hazard), is the one which we criticized in our 2nd report “Analyse critique des formules déterministes”.

In short, we repeat our conclusion of our second report : yes, probabilistic formulas should be used, but these probabilistic formulas should be based upon historical data, using histograms, and not using predefined arbitrary laws, such as log-normal. The arbitrary choice of a specific law is clearly a mistake.

Here, the data at our disposal mention the coordinates of the source zone, but not the coordinates of the target source. They mention only the distance between both. So, our investigation was made with this information. It would be better, and more precise, in order to build the probability law, to specify a given square for source and a given square for target.

Activity Data

We use the information about earthquakes which was collected by various networks in the eastern Alps and was presented in the work “Bragato-2005-BSSA-AL” :

These data consider the earthquakes of the 1976 Friuli sequence (F76) and the earthquakes that occurred between January 1995 and December 2002 in the eastern Alps between 45.6° N–46.8° N, and 12° E–14° E (SEA). All the data from the F76 events (accelerograms, locations and *M*L magnitudes) are taken from the European Strong-Motion Database (ESD; Ambraseys *et al.*, 2002). They were recorded by several permanent accelerometric stations and by a few temporary ones installed after the main event (6 May 1976, *M*L 6.3).

The following map indicates the geographical locations of these earthquakes.

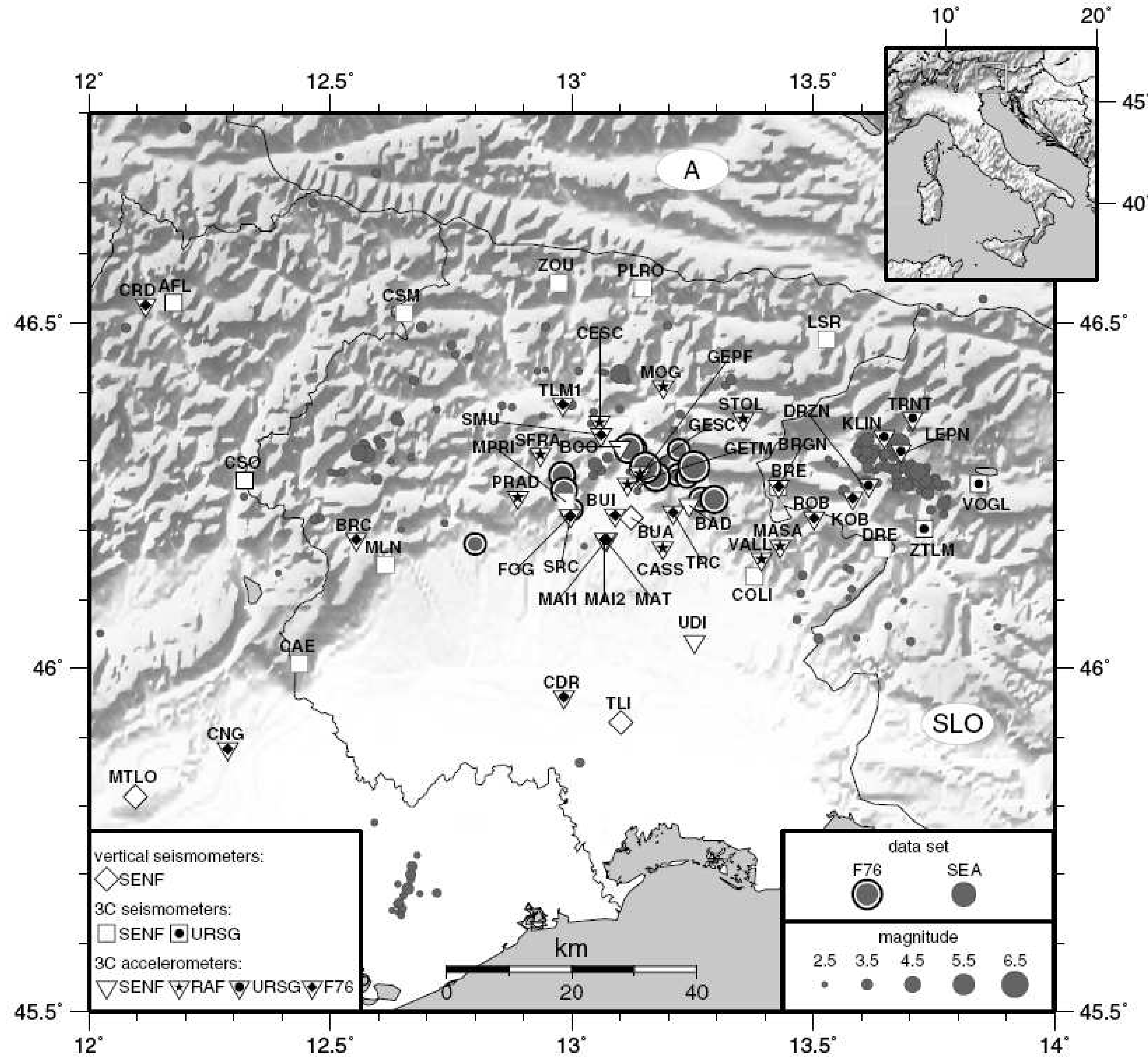


Figure 1. Epicentral map of the events considered in this study. Earthquakes of the 1976 Friuli seismic sequence are highlighted with a white border. Type and location of the RAF, SENF, and URSG stations, and those operating during the 1976 sequence, are reported as well.

So, we have at our disposal 240 data which characterize different earthquakes. For each of them, we know the date (when the earthquake was recorded), the location of the "zone source" (latitude and longitude), magnitude (M_L), depth (km), focal distance (km), PHA (Pick Horizontal Acceleration or Pick Ground Acceleration) and so on.

All data are presented in the form of the following table (we give only two lines of it) :

Table 2
Description of the Earthquakes Considered in This Work

Date (yyyy/mm/dd)	Time	Lat N (°)	Lon E (°)	Depth (km)	M_L	N. Records V	N. Records H	Distance (km) min	Distance (km) max	PVA_{max} (g)	PHA_{max} (g)
1976/05/06	19:59:06	46.277	13.239	9.0	4.5	1	1	6	6	4.16e − 02	1.57e − 01
1976/05/06	20:00:13	46.292	13.253	7.0	6.3	13	13	7	175	2.67e − 01	4.73e − 01

As we said, these data represent 9 years of observations (1976, 1995-2002).

Among all existing data we use only 4 types of information :

- Depth
- Magnitude
- Distance (here we take only *distance_max*)
- PHA

The observed magnitudes lie in the interval 2.5 – 6.3 and observed accelerations lie in the interval 0.000026 g – 0.523 g.

Using depth and distance, we compute the epicentral distance $R = \sqrt{depth^2 + distance^2}$, which represents the distance from "zone source" to "zone cible". The values of epicentral distance belong to the interval 9 km – 180 km.

In the present study we will not provide the seismotectonic zonation, and consider all territory as a single unit.

Determination of the probability law for acceleration

In our 2nd report, we mentioned the formula which is used for calculation of attenuation law between "zone source" and "zone cible" ; it follows a log-normal law :

$$P\left(A > A^* \backslash M_L, R\right) = 1 - \phi\left[\frac{\ln A^* - \overline{\ln A}}{\sigma_{\ln A}}\right] \quad (1)$$

where

– A^* is the threshold of acceleration registered in a "zone cible",
– $\overline{\ln A}$ is the average of the logarithms of accelerations in a "zone cible".

We criticized this formula, saying that the attenuation of acceleration in a soil is not known completely, so it is not really correct to describe it using deterministic laws. Conversely, we suggested to use a probability law which is based only upon the historical data themselves.

We will give here computations using both these methods.

We distinguish 3 cases :

1) We compute the probability law for the acceleration, considering the whole range of observed magnitudes and epicentral distances. In other words, what is the probability to have a certain acceleration (or to exceed some level A^*) when $M_L \in \left[2.5; 6.3\right]$ and $R \in \left[9\ km; 180\ km\right]$?

So, as we said, as total we have 240 values of accelerations which belong to the interval 0.000026 – 0.523.

We take the interval [0; 0.6] and divide it into subintervals with width 0.01. In this way we obtain 60 subintervals.

Then, we take the fist subinterval [0 – 0.01) and count how many times the observed values of PGA fell in it. In our case we found 157 such accelerations.

We take the second subinterval [0.01 – 0.02) and count the number of time when observed accelerations belong to it. We do the same computations for all subintervals.

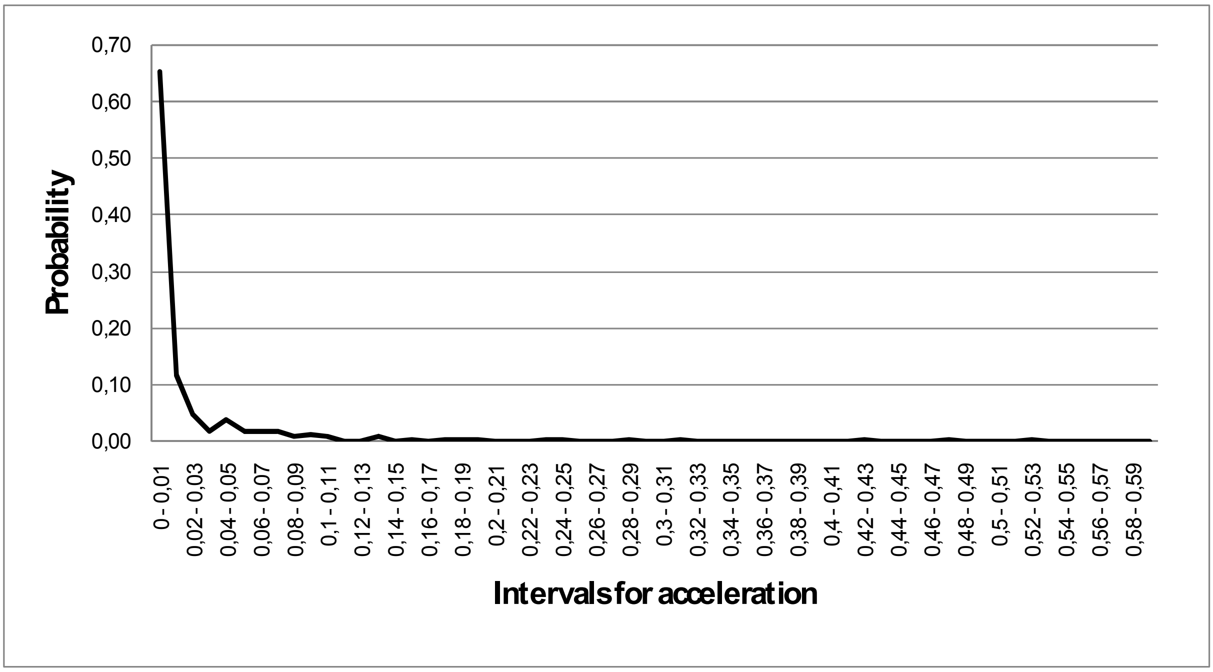
Dividing each obtained number by 240, we find the probability distribution for the acceleration :

For example, the probability that acceleration will take the values between 0 and 0.01 is equal 0.65, the probability to be in the interval [0.01 – 0.02) is equal 0.12 and so on :

Intervals for PHA	0 - 0,01	0,01 - 0,02	0,02 - 0,03	0,03 - 0,04	0,04 - 0,05	0,05 - 0,06	…
Probability	0,65	0,12	0,05	0,02	0,04	0,02	…

Figure 1 : Table of the condition probabilities.

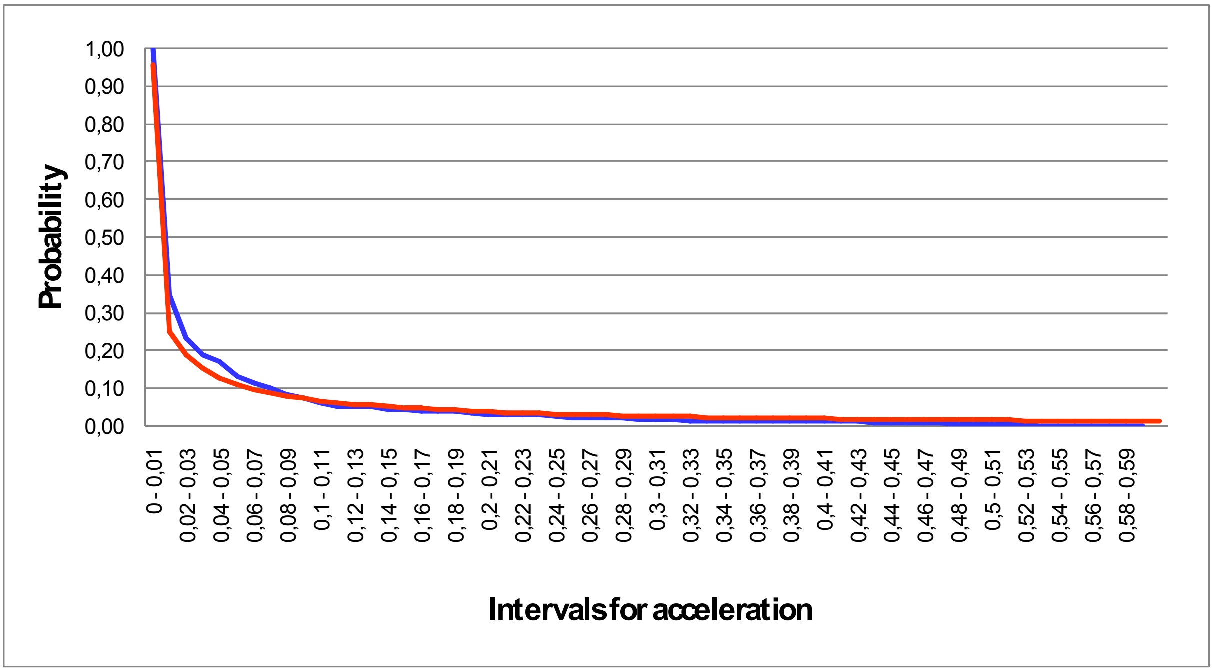
The graph of density of probability is of the form :



Graph 2 : Form of the density of probability ($M_L \in \left[2.5; 6.3\right]$ *and* $R \in \left[9\ km; 180\ km\right]$ *).*

This probability law for acceleration is valid for all earthquakes having the magnitude $M_L \in \left[2.5; 6.3\right]$ and for the epicentral distances $R \in \left[9\ km; 180\ km\right]$.

Let F be the repartition function of this probability law. The next graph shows the function $1-F$, in blue. The function $1-F$ indicates the probability to be above a certain threshold.



Graph 3 : Blue line is the proba to be above a threshold, based upon the historical data, red line is the attenuation law ($M_L \in \left[2.5; 6.3\right]$ and $R \in \left[9\ km; 180\ km\right]$).

On the other hand, we compute the attenuation law using formula (1). These computations are presented with a red line in the graph above.

This example shows that for the global case, both methods give similar results.

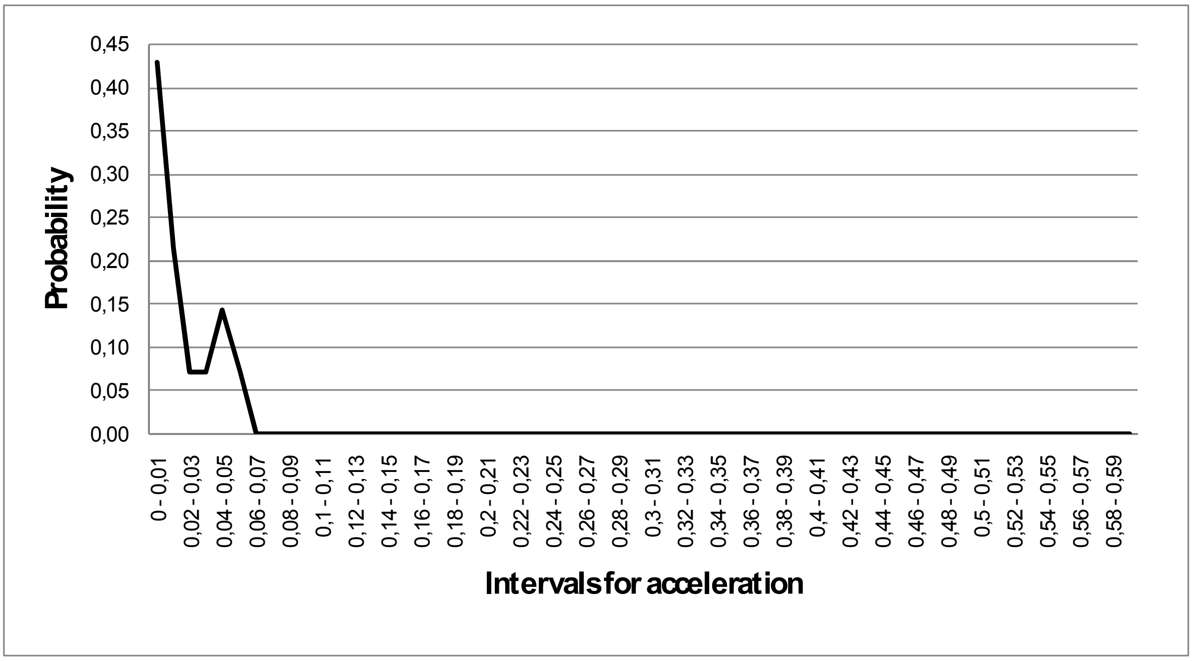
2) Using the same database, we want now to find the probability of distribution of acceleration, but only for a specified range of magnitude and distance. Let us take $M_L \in \left[2.5; 4\right]$ and $R \in \left[9\ km; 60\ km\right]$

Such a choice, for example, can be conditioned by the necessity to have the probability law for the acceleration for the weak earthquakes and for short distances from the "zone source".

Among 240 data, we select only those which have $M_L \in \left[2.5; 4\right]$ and the values of acceleration were recorded at the distance $R \in \left[9\ km; 60\ km\right]$. We found 14 data which answer these conditions.

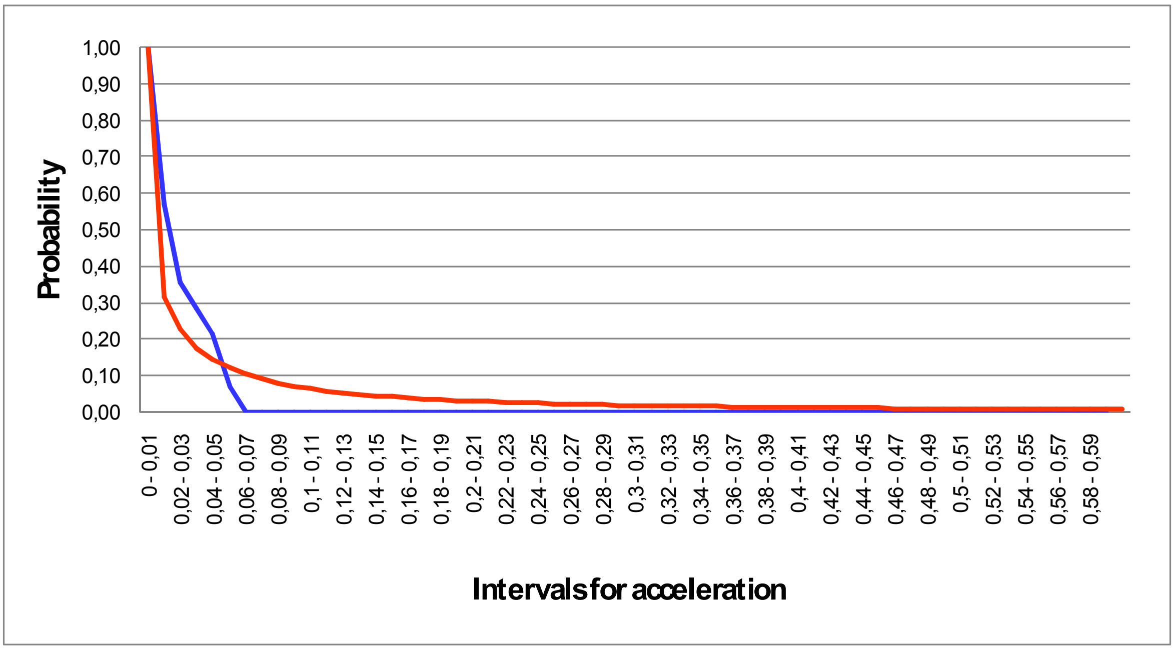
The computations are carried out the same way as before : we count how many times the vales of acceleration fall in each subinterval and divide all numbers by 14.

The probability law in this case takes the following form :



Graph 4 : Form of the density of probability ($M_L \in \left[2.5; 4\right]$ and $R \in \left[9\ km; 60\ km\right]$).

The computations of the repartition function and attenuation law give the following graph :



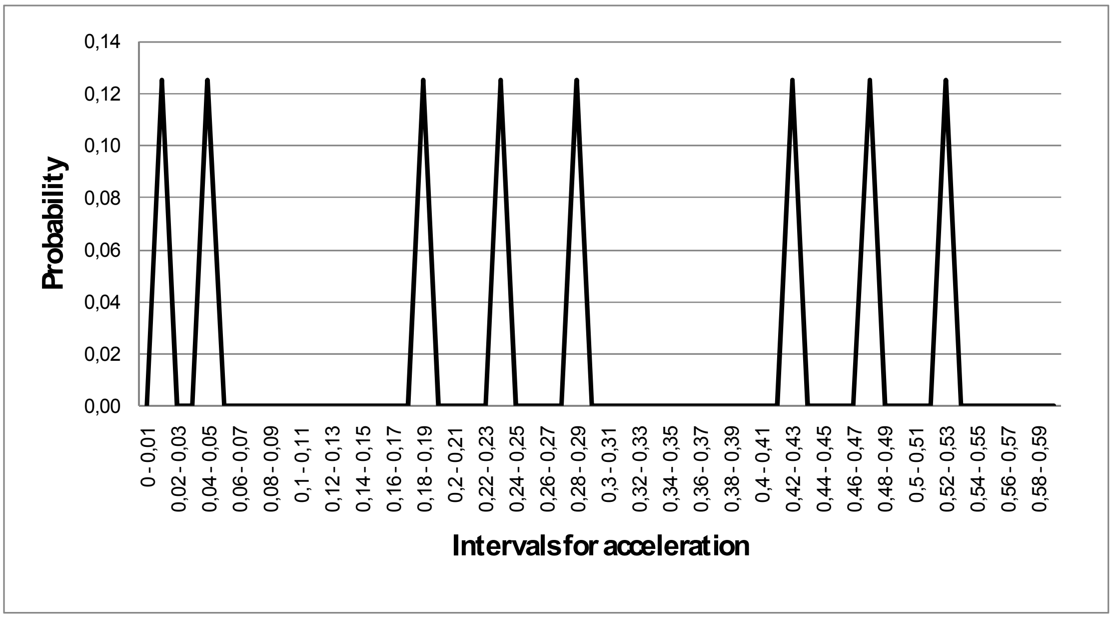
Graph 5 : Blue line is the proba to be above a threshold based upon the historical data, red line is the attenuation law ($M_L \in \left[2.5; 4\right]$ and $R \in \left[9\ km; 60\ km\right]$).

Looking at this graph, we can easily see that the attenuation law does not describe well the behavior of acceleration : it underestimates the probability that acceleration will belong to the interval [0; 0.05) and conversely the probability that the acceleration will be more than 0.05 g is overestimated.

3) Finally, we consider the case when $M_L \in [4; 6.3]$ and $R \in [100\ km; 180\ km]$. This corresponds to the case of strong earthquakes at long distances.

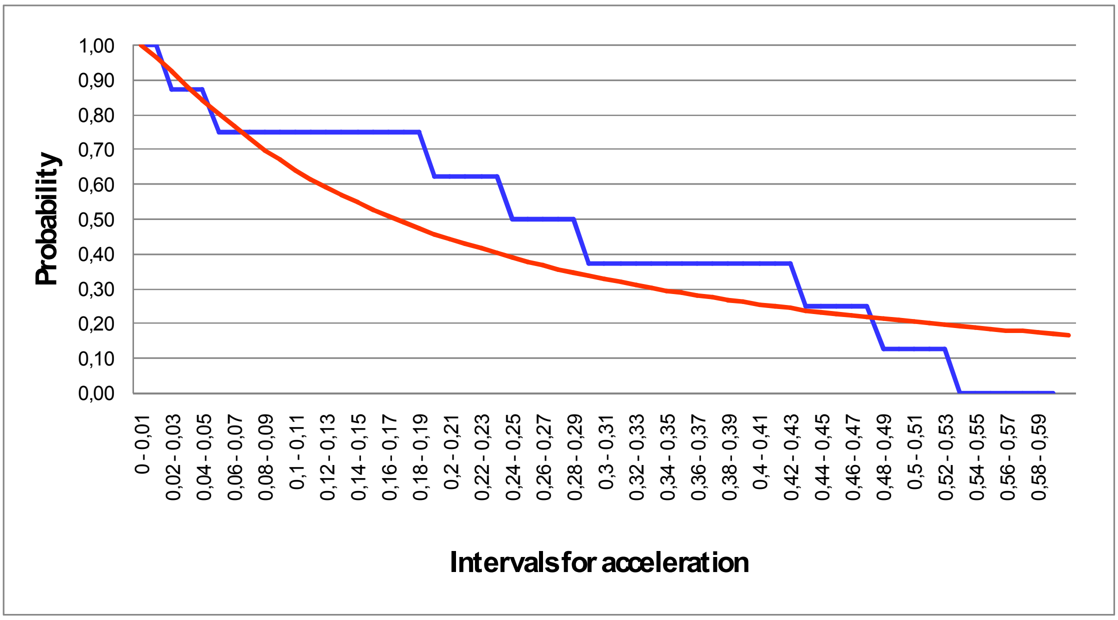
Again, we select only those observations which satisfy these conditions. In this case, we found 8 data of accelerations.

All 8 values fall in 8 different intervals, so the probability law of acceleration takes the following form:



Graph 6 : Form of the density of probability ($M_L \in [4; 6.3]$ and $R \in [100\ km; 180\ km]$).

The proba to be above a threshold and the attenuation law are the following :



Graph 7 : Blue line is the proba to be above a threshold based upon the historical data, red line is the attenuation law ($M_L \in [4; 6.3]$ and $R \in [100\ km; 180\ km]$).

Remark

In our 2nd report, we also gave an example showing that a probabilistic approach based upon historical data gives a much better result than the use of an artificial law. This example dealt with the probability of occurrence of an earthquake, with a certain magnitude, in a "zone source". It concerns all French earthquakes, between 07-01-2000 and 2007. We reproduce it here :

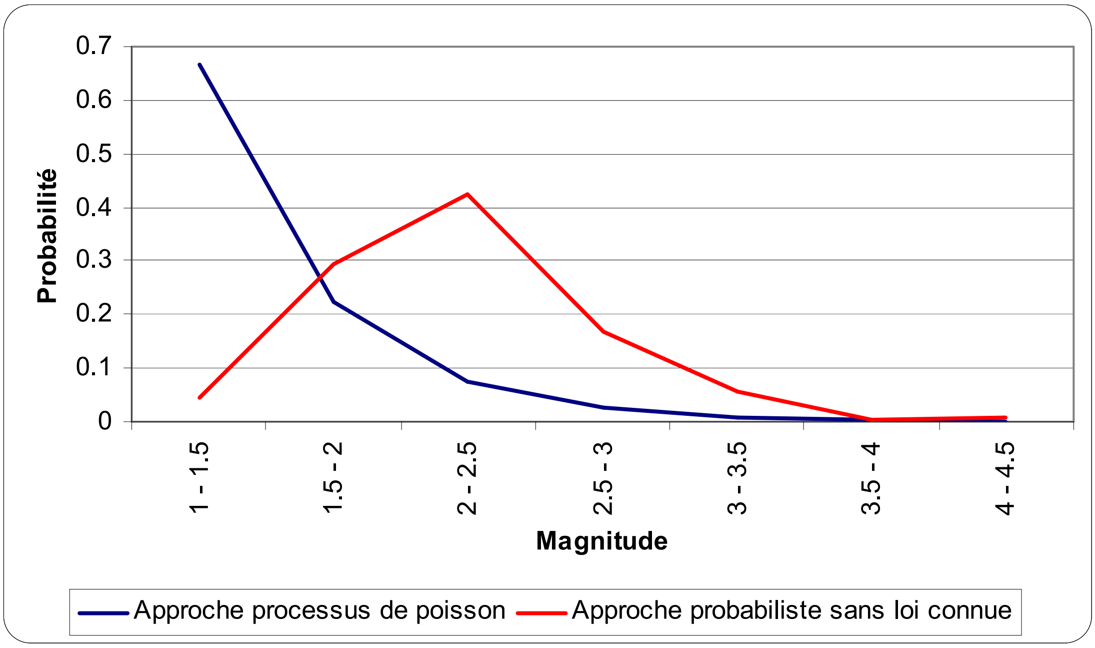


Figure 8 : Comparison between Poisson approach with the probabilistic one.

Conclusion

These examples show that the artificial law does not always represents well the physical process (in our case : the propagation of acceleration). The use of probability laws, based only upon the historical data, looks more favorable.