

# Probabilistic Safety Assessment and Reliability Engineering: Reactor Safety and Incomplete Information

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**Abstract** – *The protection-system of GEN III+ reactors partially relies on self-powered neutron detectors (SPNDs), in-core devices measuring the neutron flux. The goal of this paper is to estimate the robustness and reliability of those detectors for surveillance purposes, using mathematical models. The results of research might serve as the basis for decision-making regarding future development and amelioration of the system.*

## I. INTRODUCTION

The GEN-III NPPs (Nuclear Power Reactors) rely on a wide and rich instrumentation, including the Self-Powered-Neutron-Detectors - SPND - which, as it can be the case for cobalt ones [REF. 1], are adopted as part of a protection system for safety purposes. Those devices, also called collectrons, provide indirect measurements of the neutron flux assembly-wise averaged values, which are used, through a suitable algorithm, to reconstruct the core hot spot (the place in the core where the power peaks).

For operation and safety reasons, it is worth quantifying the increase in the uncertainty affecting the reconstructed hot spot value, when one or more SPNDs fail. More precisely, the aim is to determine if, in some cases, the uncertainty affecting the reconstructed power value in the hot spot exceeds acceptable, pre-defined limits.

Two major difficulties need to be addressed:

- This is an inverse problem, of non-linear nature;
- Due to the physical features of the multiplying system, the collectrons receive directly only a very small

fraction of the neutrons emitted in the hot spot; actually, most of neutrons reaching the SPNDs are the descendants of the descendants... of the descendants of the “primitive” information-carriers.

Accordingly, the mathematical approach has been subdivided into three steps:

- Direct problem. Establishing a model for the propagation of neutrons. For a given position and intensity of the hot spot, how many neutrons originating there and how many of their direct descendants will be collected in each SPND? The method we adopted consists in discretizing the core into roughly regular parallelepipeds and assessing the emission of neutrons from one parallelepiped to its nearest neighbours. In a first approximation, the emission is assumed to be isotropic and the neutrons are assumed to be mono-energetic;
- Inverse problem. Setting-up a probabilistic method allowing the reconstruction of the hot spot value, relying on SPND measurements. This method rests upon transfer functions, which provide the asymptotic contribution of each individual collectron to the intensity of the hot spot. We build the joint probability law, which allows us to

appreciate the quality of the reconstruction. We use this method on many samples of SPND measurements, which enables us to determine how frequently the uncertainty of the reconstruction is not acceptable.

- Global approach. The adaptation of the probabilistic approach called EPH (Experimental Probabilistic Hyper-surface) for the current target settings. This method is used to handle the uncertainties relying on a general principle of maximal entropy (minimal information).

Both methods can be adopted to simulate the breakdown of one or several SPNDs; thus the sensitivity of the uncertainty of the hot spot reconstruction to the breakdowns can be determined. The larger this uncertainty is, the higher the risk will be for the protection system to be unreliable.

In a first approach, neutrons are assumed to be mono-energetic. In a second approach, this model has been extended by accounting for two types of neutron families (as usual, the fast and thermal neutrons).

The paper is intended to describe the above mentioned methodologies, present their preliminary application to a GEN III-type reactor core, draw some general conclusions on the reliability of the protection system adopting such measurement devices, and define the main steps for further improvements and applications

## II. ASSUMPTIONS

The collectrons - SPNDs - are used to monitor the propagation of neutrons in the reactor core and provide information on the hot spot. They are arranged inside the core in 12 vertical arrays, each containing 6 of them. In total, 72 devices measure gamma rays originating from neutron capture.

The objective of the work is to estimate the deterioration of the quality of reconstruction of the hot spot value, when one or more SPNDs are considered as unreliable.

The approach is based on several assumptions about the core configuration and the neutrons propagation:

- Arrays are modelled as roughly 4-meter-high and 21.5-cm-long parallelepipeds, discretized into 7 vertical three-dimensional 60-cm-cells. SPNDs are not located exactly at the centre of each mesh; this approximation is considered as inconsequential. Moreover, it is assumed that the presence of a SPND in a cell has no impact on its neutronics behaviour.

- The zero-burn-up assembly-wise infinite multiplication factors have been normalised in order to guarantee a roughly critical condition;
- The fission neutrons are isotropic; moreover the neutrons do not have privileged directions. Accordingly, the leakage is assumed to be proportional to the surfaces in contact. Likewise, the capture of the neutrons by the collectrons is also isotropic;
- The hot spot emits neutrons continuously;
- The core boundary condition is a non-reflective black absorber (represented by a medium the multiplication factor of which is zero).
- In a first approximation, neutrons are supposed to be monochromatic: namely they have equal kinetic energy and belong to a single family.
- In the implemented model, the energy of neutrons is taken into account the following way: they are divided into two families, with the averaged simplified features below:
  - a) *Fast neutrons* (high energy): they do not interact with the atoms owing to their speed; thus, they do not participate in the fission process and so, they are non-fission-engendering neutrons. Because of the "shocks" with the nuclei of the medium, the fast neutrons slow down, lose their energy and become thermal neutrons. This process is called slowing-down. We assume that the fast neutrons which leave one cell are fully slowed-down when arriving in the neighbouring one.
  - b) *Thermal neutrons* (low energy): the speed of such neutrons is low enough to create fissions and give birth to fast neutrons. The amount of fast neutrons created by fission depends on the infinite multiplication factor of the medium where the fission takes place. The process of emission of fast neutrons after the fission is assumed to be instantaneous and localized in the same cell.
- The hot spot emits only fast neutrons;
- The SPNDs are sensitive to the thermal neutrons only.

## III. DIRECT PROBLEM

The first step of the work consists in modelling the direct problem: for a given position and intensity of the hot

spot in a cell of the core, how many neutrons does each SPND receive?

The mathematical difficulty to address comes from the fact that, due to the multiplying features of the media, most of the neutrons collected by the SPNDs do not originate from the hot spot. Actually, they are the descendants of the descendants... of the descendants of neutrons emitted by the hot spot.

The method we developed uses a cross, which is moved across the reactor, and calculates the neutrons propagation at every step. This cross is composed of one cell with its neighbours (4 neighbours in 2D, 6 neighbours in 3D). We describe the method by using relative values.

The cross is first placed at the hot spot. At  $T = 1$ , this point emits 1 neutron, spreading to the 4 neighbouring regions with equal probability  $1/4$  at  $T = 2$ . The value of the quantity of neutrons  $Q$  contained in each cell of the core (except the hot spot) at the step  $T = N$  is determined by induction:

$$Q_{i,j,N} = \frac{1}{4} \times (\lambda_{i-1,j} Q_{i-1,j,N} + \lambda_{i+1,j} Q_{i+1,j,N} + \lambda_{i,j-1} Q_{i,j-1,N} + \lambda_{i,j+1} Q_{i,j+1,N}) \quad (1)$$

where  $(i, j)$  are the coordinates of the cell in the core,  $\lambda_{i,j}$  is the infinite multiplication factor of the material  $(i, j)$ . The quantity of neutrons in the hot spot cell at the step  $T = N$  is calculated the same way, except that the hot spot emits 1 additional neutron at each step.

The principle is generalized and extended to the 3D-model, with a 3D-region composed of 7 parallelepipeds. The method is convergent: the quantity of neutrons received by each cell is stabilized after a few hundreds of runs for the 2D model, a few thousands of runs for the 3D model.

This direct model has been improved by the consideration of the uncertainties in the materials composition, as well as in the isotropy of the neutrons emission from a cell; the model is thus closer to the physical reality. An uncertainty of  $\pm 10\%$  is assigned to the probability for a neutron to head for one of the neighbouring cells and  $\pm 2.5\%$  on the value of the multiplication factor of the cell material. These uncertainties are independent from one cell to another.

Thus, a more realistic representation of the propagation is obtained because probability laws are now available.

#### IV. RECONSTRUCTION OF THE HOT SPOT VALUE USING TRANSFER FUNCTIONS

The solution of the inverse problem relies upon the transfer functions. The method aims at answering the following question: knowing the quantity of neutrons collected by SPNDs at a time, what is the probability law for the hot spot intensity? This allows the quantification of the uncertainty on the intensity reconstruction.

The transfer function from the hot spot to the SPND  $i$  is defined by the ratio between the intensity of the hot spot and the quantity of neutrons collected by the SPND. For instance, in the deterministic case, for 10000 neutrons emitted in the hot spot, SPND 1 collects 209 neutrons. The transfer function from the hot spot to SPND 1 is thus  $\frac{10000}{209}$ . As a result, we can deduce that, for instance, if the collectron 1 measures an intensity of 50 neutrons, then the hot spot intensity is 2392 neutrons.

When taking into account the uncertainties linked to the neutrons emission, this transfer function is a random variable, represented by its probability law. We could say (for instance): given that SPND 1 measures an intensity of 50 neutrons, the hot spot intensity is 2200 with probability  $1/3$ , and 2500 with probability  $2/3$ .

Therefore, the value collected in each SPND allows us to obtain probability laws of the hot spot intensity: the density of the intensity reconstructed with the SPND  $i$  is called  $f_i$ .

The combination of the laws given by each collectron enables a better knowledge of the hot spot intensity. Assuming the devices to be fully independent from each other, the final density of the hot spot value is:

$$f_{HS} = \frac{1}{C} \times f_1 \times \dots \times f_{N_{collectrons}} \quad (2)$$

where  $C$  is a normalization coefficient, and  $N_{collectrons}$  is the number of collectrons.

The independence assumption means that the errors made by a measurement device do not influence the errors made by other ones, which is actually the case.

The final aim is not only to obtain a reconstruction method of the hot spot intensity, but also to characterize the quality of the reconstruction, in particular in the case of a breakdown of one (or several) SPND (s): is the uncertainty

on the reconstructed average value low enough to guarantee the reactor safety?

At this point, interest is focused on the 90% confidence interval: it quantifies the uncertainty around the reconstructed average value. This confidence interval is limited by the 5% and 95%-percentiles, defined as follows: the probability for the hot spot value to be higher than the 5% percentile value is 0.95.

We compare the percentiles value with the expected value: if the relative deviation is higher than 5%, then the IRSN will consider the reconstruction as being poor, in the sense that our knowledge of the hot spot intensity is not sufficient enough to ensure safety.

This method can be applied for the simulation of SPND failures: the impact of the failure on the uncertainty of the hot spot reconstruction can thus be quantified with an indicator.

#### IV.A. Results

Both in 2D and 3D, whenever one SPND is lost, the corresponding uncertainty on the reconstructed value of the hot spot increases.

Unfortunately, the method strongly couples the uncertainty on the reconstructed values to the quantities measured by the SPNDs, which is a drawback for its reliability and generality.

To avoid this problem, in the 2D model, the reconstruction is applied to a large number of SPND measurements (2,000). Then, the histogram of the uncertainties given by the successive breakdowns of the 12 SPNDs is built; which enables to determine which SPND is more relevant to the hot spot intensity reconstruction reliability.

Table I provides the reconstruction uncertainty averages, as well as the probability for the uncertainty to be higher than or equal to 5%. Both indicators allow assessing the system reliability.

Whichever SPND breaks down, the law is "shifted" to the right by the breakdown. This means that the uncertainty of the reconstruction is higher when a SPND breaks down than when all the 12 devices are available. This result is straightforward: the quantity of information is higher when all collectrons work simultaneously.

Moreover, the results are consistent with the reactor topography: on average, the reconstruction is more

uncertain when one of SPNDs close to the hot spot breaks down, and the probability for the uncertainty of the reconstruction to exceed the target threshold (5%) shows-up higher when the SPNDs closest to the hot spot are failing (SPNDs 4, 7, 9 and 11). Accordingly, these devices are more relevant to the information on the hot spot than the ones located far away.

TABLE I  
 Quality of the Reconstruction Depending on SPNDs  
 Breakdown

	Average uncertainty of the hot spot intensity reconstruction	Probability of the uncertainty to be higher than or equal to 5%
12 SPNDs working	4,03%	5,72E-04
Breakdown of SPND 1	4,15%	1,14E-03
Breakdown of SPND 2	4,21%	1,72E-03
Breakdown of SPND 3	4,13%	1,72E-03
Breakdown of SPND 4	4,21%	3,43E-03
Breakdown of SPND 5	4,18%	1,14E-03
Breakdown of SPND 6	4,23%	1,14E-03
Breakdown of SPND 7	4,26%	2,86E-03
Breakdown of SPND 8	4,16%	1,14E-03
Breakdown of SPND 9	4,21%	2,86E-03
Breakdown of SPND 10	4,13%	5,72E-04
Breakdown of SPND 11	4,21%	2,29E-03
Breakdown of SPND 12	4,15%	5,72E-04

In conclusion, the sensitivity of the hot spot reconstruction to the SPND availability and position is confirmed, as expected. Nevertheless, the increase of the uncertainty in the worst case of the loss of a full column does not seem too large, compared to its already quite high value when all the detectors are available. ( $5.7 \times 10^{-4}$ ).

In 3D, the reconstruction procedure is applied to a smaller number of SPND measurements (due to calculation time, about 20 cases have been studied). The histogram of uncertainties obtained for successive breakdowns of the 72 SPNDs is then built-up.

Results are similar to those obtained in 2D: when the breaking down device is located next to the hot spot, the reconstruction shows a poorer quality. However, whichever SPND breaks down, the reconstruction is better than in 2D: the uncertainty never exceeds 2%; so that the probability to be higher than the fixed threshold is closer to zero.

## V. PROBABILISTIC HYPERSURFACE

SCM has developed a mathematical model called EPH, which is used in order to reconstruct or predict the information resting upon data already given or calculated. This method relies upon a physical principle of maximal entropy: no artificial assumptions are made. The key point is the “propagation” of the information from measured points (incoming data) to any unknown ones with probability laws depending on the distance.

The results are presented in the form of a collection of probability densities. Such a density takes the form of a Dirac function at the measurement point location (the value is known precisely), and becomes less and less concentrated when moving further away from it. At the end of the process, the so-obtained individual laws are recombined to get a single one depending on the distance of the target-point from the measurement points.

Adoption of this methodology allows reconstructing the hot spot value in a global manner, without relying upon the quantities of neutrons recorded by each device.

The EPH has been developed and significantly implemented in order to address current problems in nuclear safety [REF 2]. In the present case, the major difficulty is to handle the non-homogenous multiplying medium.

The main principles of the method are the following:

Taking into account the uncertainties connected with the neutrons emission and according to the transfer function, the probability law for reconstruction is deduced for each independent device. These collectron-dependent probability laws characterise the likelihood of each collectron to reconstruct the intensity of the hot spot subject to their positions.

EPH provides a way to recombine them into a single probability law taking into account the deterioration of information with the distance: the larger the distance from the hot spot is, the flatter the probability law for the SPND will be. The speed of deterioration is governed by a general principle of maximum entropy: at the core edge the reconstruction has the worst quality (uniform law), moving towards the measure points (namely, approaching the SPND) it becomes more and more precise assuming the form of a Gaussian function:

$$p_{n,j}(X) = \frac{\tau}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(j - C_n)^2}{2\sigma^2}\right) \quad (3)$$

$$\text{with } \sigma = \frac{\lambda d_n}{\sqrt{2\pi e}} \quad (4)$$

where  $X$  is the hot spot,  $j$  is the amount of neutrons,  $C_n$  is the value associated with the  $n$ -th collectron (in our case, it is not a precise value, but a probability law for reconstruction mentioned above); the parameter  $\lambda$  is the coefficient connected with the entropy and finally  $d_n$  is the distance which is presented as a combination of all possible paths connecting the hot spot and the  $n$ -th collectron multiplied by their probability:

$$d_n \text{ Collectron}_n, \text{HotSpot} = \text{path}_1 p_1 + \dots + \text{path}_w p_w \quad (5)$$

The resulting probability is given by the formula:

$$p_j(X) = p_{1,j} \times \dots \times p_{n,j} \quad (6)$$

### V.A. Quality of the Reconstruction in 2D

First of all, we assume that the hot spot emits a given amount of neutrons. Then, assuming a random deviation from  $1/4$  within a range of  $\pm 10\%$  they are propagated in the space of the reactor core and the amount received by each collectron is counted off (in the 2D description, only 12 of them are described). Simulating this process many times, samples are obtained which allow constructing the probability laws for each device. Then, these laws are combined according to EPH rules.

The resulting probability laws are assessed with the “relative variances”:

$$V_1 = \frac{E[X] - Q_{5\%}}{E[X]}, \quad V_2 = \frac{Q_{95\%} - E[X]}{E[X]} \quad (7)$$

### V.B. Obtained Result:

Finally, the reconstruction of the hot spots which are located near the reactor core centre is made.

If all 12 collectrons are assumed to be reliable and participate in the hot spot reconstruction, then the probability law has the following characteristics:

$$\text{central } V_1^{(12\text{captors})} = 3,8\% , \quad \text{central } V_2^{(12\text{captors})} = 3,8\%$$



### VI.B. Results Obtained with the Use of Transfer Functions

In 2D, the results obtained prove to be a little bit more penalizing than in the previous model: the average uncertainties are a little more important (the average uncertainty with one collectron missing is 3.7%, while it was 4.2% with mono-kinetics neutrons).

Moreover, the probability for the uncertainty of the reconstructed value of the hot spot intensity to be higher than the threshold is often close to zero, except when the breakdown affects the collectrons which are located very close to the hot spot. In these cases, the probability to exceed the threshold is the same as in the previous model ( $2E^{-3}$  or  $5E^{-3}$ , depending on which SPND is missing). These values look quite high, which could ask for complementary, more accurate, investigations.

TABLE II  
 Quality of the Reconstruction Depending on SPNDs  
 Breakdown Taking into Account Two Types of Neutrons

	Average uncertainty of the hot spot intensity reconstruction	Probability of the uncertainty to be higher than or equal to 5%
12 SPNDs working	3,56%	0
Breakdown of SPND 1	3,70%	0
Breakdown of SPND 2	3,74%	0
Breakdown of SPND 3	3,66%	0
Breakdown of SPND 4	3,76%	0
Breakdown of SPND 5	3,70%	0
Breakdown of SPND 6	3,77%	0
Breakdown of SPND 7	3,82%	5,31 E-03
Breakdown of SPND 8	3,69%	0
Breakdown of SPND 9	3,75%	2,65 E-03
Breakdown of SPND 10	3,68%	0
Breakdown of SPND 11	3,76%	2,65 E-03
Breakdown of SPND 12	3,69%	0

### VI.C. Results Obtained with the EPH Method

Using EPH, the reconstruction of the hot spot is performed taking into account the new conditions. The quality of the reconstruction obtained widely agrees with the previous conclusions.

### VI.D. Propagation in 3D

The multi-energy model of neutrons propagation in 3D appears to be much more complex because of the non equilateral discretization of the core: each three-dimensional cell takes the form of parallelepiped with the height (60 cm) which is almost 3 times the side of the square (21.5 cm). This information has been embedded in the model assuming a vertical passage equal  $2T$  and horizontal one equal  $T$ .

This system is also convergent within 0.01sec.

The distribution of neutrons takes a different form due to the new conditions regarding the speed of propagation in each direction.

### VI.I. Results Obtained with the EPH Method

The characteristics of the probability law of reconstruction were changed. They indicate a slight degradation of the quality:

$$\text{central } V_1^{(72\text{captors})} = 1,19\% , \quad \text{central } V_2^{(72\text{captors})} = 1,06\%$$

Nevertheless, as in the previous investigation, the results obtained do not differ significantly from the 2D ones, which demonstrates that the mono-energetic system is rather efficient, despite the simplifications.

## IV. CONCLUSIONS

Two different and independent mathematical approaches have been adopted in order to investigate the reliability and robustness of the in-core SPND-based protection system of the GEN-III type reactors, against the detector failures. The results show a quite good convergence and the same trends.

As expected, the sensitivity of the hot spot reconstruction to the SPND availability and position is confirmed. Nevertheless, the increase of the uncertainty in the worst case - loss of a full column of SPNDs due, e.g., to an electrical connection failure - does not seem too large, compared to its already quite high value when all the detectors are available.

As expected, the sensitivity appears to be lower in 3D, mainly due to a smaller individual contribution of the detectors to the hot spot value reconstruction.

Finally, more precise calculations accounting for the energy dependence of neutrons do not change the results

significantly, except when the breakdown affects the collectrons which are located very close to the hot spot. These cases could require complementary and more accurate investigations.

#### REFERENCES

1. Proceedings of the International Symposium on Radiation Physics-ISR7 N°7, “*Some studies on cobalt and vanadium self powered neutron detectors developed by ECIL*”, Jaipur, INDE (1997), 1998, vol. 51, n° 4-6 (409 p.) (2 ref.), pp. 453-454P., S. Rao, A.K. Manta, S. Rao, S.M. Tripathi & S.C. Misra.
2. Olga Zeydina «*Méthodes probabilistes pour l'analyse des incertitudes liées à la sûreté des réacteurs nucléaires. L'Hypersurface Probabiliste : construction générale et applications* », April 2007.  
[http://www.scmsa.com/RMM/IRSN\\_SCMSA\\_EPH4.pdf](http://www.scmsa.com/RMM/IRSN_SCMSA_EPH4.pdf)